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6. Selected Radioactive Isotopes
7. Useful Information
8. Glossary of Key Symbols and Notation

Preface

Welcome to *College Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it’s our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax Resources

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Format

You can access this textbook for free in web view or PDF through openstax.org, and in low-cost print and iBooks editions.

About *College Physics*

College Physics meets standard scope and sequence requirements for a two-semester introductory algebra-based physics course. The text is grounded in real-world examples to help students grasp fundamental physics concepts. It requires knowledge of algebra and some trigonometry, but not calculus. *College Physics* includes learning objectives, concept questions, links to labs and simulations, and ample practice opportunities for traditional physics application problems.

Coverage and Scope

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Chapter 1: Introduction: The Nature of Science and Physics
Chapter 2: Kinematics
Chapter 3: Two-Dimensional Kinematics
Chapter 4: Dynamics: Force and Newton's Laws of Motion
Chapter 5: Further Applications of Newton's Laws: Friction, Drag, and Elasticity
Chapter 6: Uniform Circular Motion and Gravitation
Chapter 7: Work, Energy, and Energy Resources
Chapter 8: Linear Momentum and Collisions
Chapter 9: Statics and Torque
Chapter 10: Rotational Motion and Angular Momentum
Chapter 11: Fluid Statics
Chapter 12: Fluid Dynamics and Its Biological and Medical Applications
Chapter 13: Temperature, Kinetic Theory, and the Gas Laws
Chapter 14: Heat and Heat Transfer Methods
Chapter 15: Thermodynamics
Chapter 16: Oscillatory Motion and Waves
Chapter 17: Physics of Hearing
Chapter 18: Electric Charge and Electric Field
Chapter 19: Electric Potential and Electric Field
Chapter 20: Electric Current, Resistance, and Ohm's Law
Chapter 21: Circuits and DC Instruments
Chapter 22: Magnetism
Chapter 23: Electromagnetic Induction, AC Circuits, and Electrical Technologies
Chapter 24: Electromagnetic Waves
Chapter 25: Geometric Optics
Chapter 26: Vision and Optical Instruments
Chapter 27: Wave Optics
Chapter 28: Special Relativity

Chapter 29: Introduction to Quantum Physics
Chapter 30: Atomic Physics
Chapter 31: Radioactivity and Nuclear Physics
Chapter 32: Medical Applications of Nuclear Physics
Chapter 33: Particle Physics
Chapter 34: Frontiers of Physics
Appendix A: Atomic Masses
Appendix B: Selected Radioactive Isotopes
Appendix C: Useful Information
Appendix D: Glossary of Key Symbols and Notation

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Key Features

Modularity

This textbook is organized as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers’ attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado. There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a Glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online, every other problem includes an answer that students can reveal immediately by clicking on a “Show Solution” button.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem’s solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer.

Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Additional Resources

Student and Instructor Resources

We’ve compiled additional resources for both students and instructors, including Getting Started Guides, an instructor solution manual, and PowerPoint slides. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

Partner Resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

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Introduction to Science and the Realm of Physics, Physical Quantities, and Units

class="introduction"

Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature’s apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for

Astrophysics
)

What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people’s regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone ([link]). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

The Apple "iPhone" is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See [link] and [link].) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

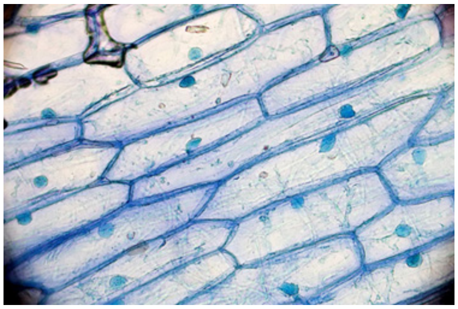
Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes ([link] and [link]). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

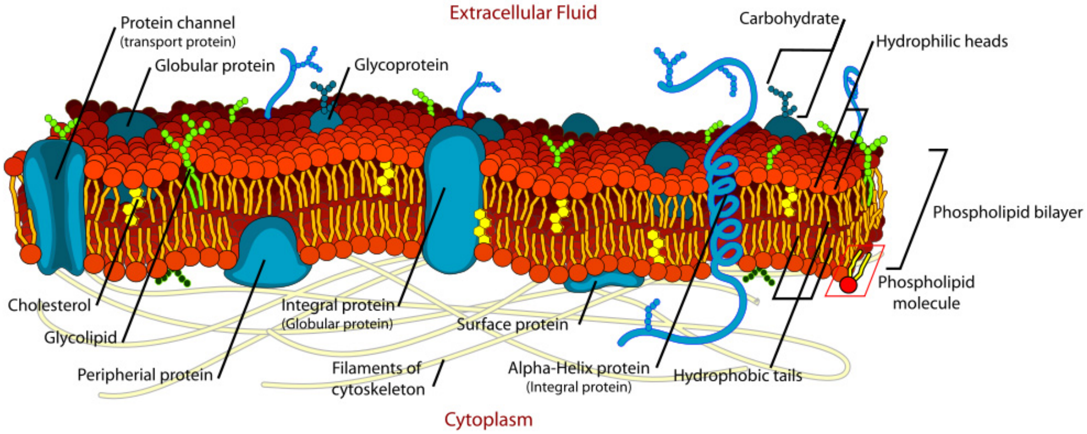
The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



Physics, chemistry,

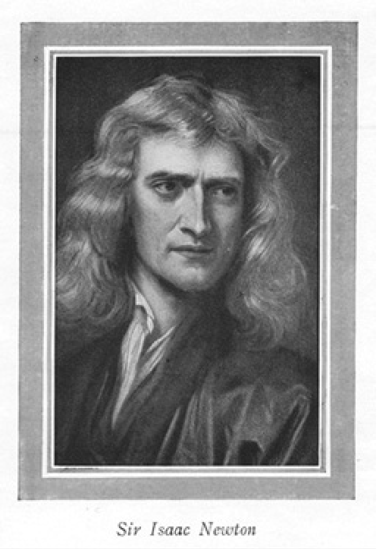
and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)


Extracellular Fluid
Protein channel
(transport protein)
Globular protein
Glycoprotein
Carbohydrate
Hydrophilic heads
Phospholipid bilayer
Phospholipid
molecule
Cholesterol
Glycolipid
Peripherial protein
Integral protein
(Globular protein)
Surface protein
Filaments of
cytoskeleton
Alpha-Helix protein
(Integral protein)
Hydrophobic tails
Cytoplasm

An artist’s rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See [link] and [link].) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Marie Curie (1867–1934) sacrificed

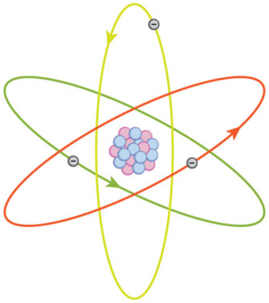
monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See [link].) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton’s theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton’s second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal’s principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.



What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Note:

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

Note:

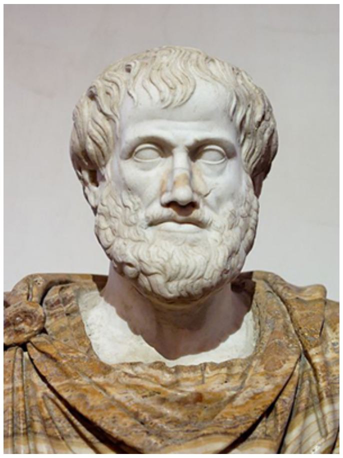
The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

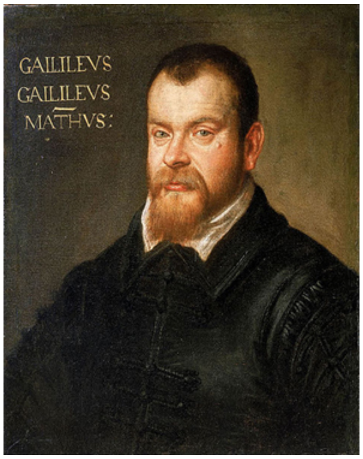
The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See [link], [link], and [link].) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

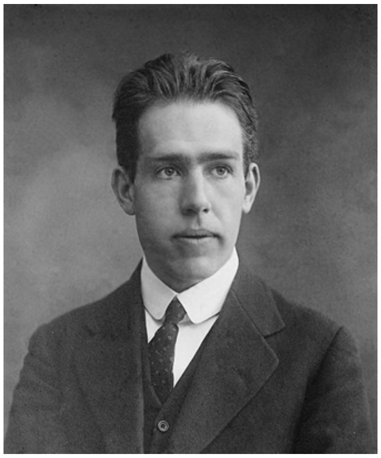


Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow

(2006)/Ludovisi Collection)


GALILEVS
GALILEVS
MATHVS:

Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)



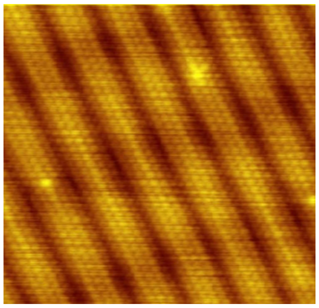
Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom’s properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

Note:

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Exercise:

Check Your Understanding

Problem:

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution:

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

Note:
PhET Explorations: Equation Grapher
Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.
https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html

Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

Conceptual Questions

Exercise:

Problem:

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

Exercise:

Problem: How does a model differ from a theory?

Exercise:

Problem:

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

Exercise:

Problem: What determines the validity of a theory?

Exercise:

Problem:

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

Exercise:

Problem:

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

Exercise:

Problem:

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

Exercise:

Problem: When is it *necessary* to use relativistic quantum mechanics?

Exercise:

Problem:

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

Glossary

classical physics
physics that was developed from the Renaissance to the end of the 19th century

physics
the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

model
representation of something that is often too difficult (or impossible) to display directly

theory
an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

law
a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence

and repeated experiments

scientific method
a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

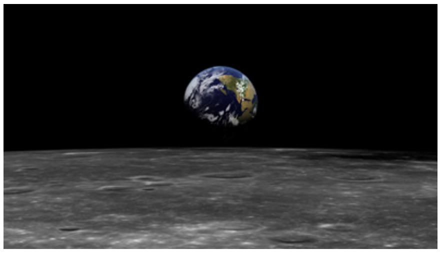
modern physics
the study of relativity, quantum mechanics, or both

relativity
the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

quantum mechanics
the study of objects smaller than can be seen with a microscope

Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

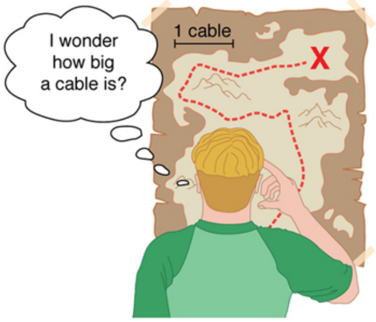


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See [link].)


I wonder how big a cable is?
1 cable
X

Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

[link] gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

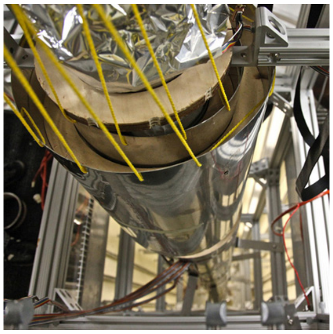
Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth’s rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See [link].) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



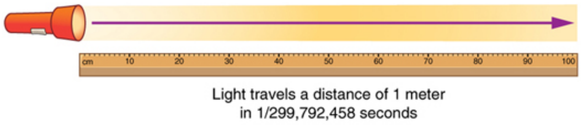
An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See [link].) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.



The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in Introduction to Electric Current, Resistance, and Ohm's Law when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. [link] gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}\ \text{m}$, while the diameter of the Sun is on the order of $10^9\ \text{m}$.

Note:
The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

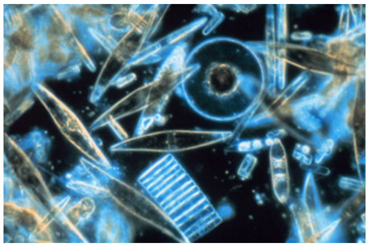
Prefix	**Symbol**	**Value[footnote] See Appendix A for a discussion of powers of 10.**	**Example (some are approximate)**			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons

Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (some are approximate)			
deka	da	10^1	dekagram	dag	10^1 g	teaspoon of butter
—	—	10^0 (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

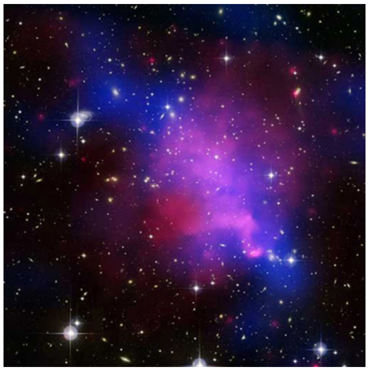
Metric Prefixes for Powers of 10 and their Symbols

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in [link]. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See [link] and [link].)



Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)



Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:
Equation:

$$80\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}} = 0.080\text{ km}.$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click [link] for a more complete list of conversion factors.

Lengths in meters		**Masses in kilograms (more precise values in parentheses)**		**Times in seconds (more precise values in parentheses)**	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $\left(9.11 \times 10^{-31}\text{ kg}\right)$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $\left(1.67 \times 10^{-27}\text{ kg}\right)$	10^{-22}	Mean life of an extremely unstable nucleus

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $(8.64 \times 10^{4} \text{ s})$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $(3.16 \times 10^{7} \text{ s})$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $(7.35 \times 10^{22} \text{ kg})$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $(5.97 \times 10^{24} \text{ kg})$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $(1.99 \times 10^{30} \text{ kg})$		

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Approximate Values of Length, Mass, and Time

Example:
Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

Equation:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.

Equation:

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}.$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

Equation:

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

Equation:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

Equation:

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3,600\text{ s}} \times \frac{1,000\text{ m}}{1\text{ km}},$$

Equation:

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Accuracy, Precision, and Significant Figures will help you answer these questions.

Note:

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Exercise:
Check Your Understanding

Problem:

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution:

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Exercise:
Check Your Understanding

Problem:

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution:

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

Exercise:

Problem: Identify some advantages of metric units.

Problems & Exercises

Exercise:

Problem:

The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

Solution:

a. $27.8\ \text{m/s}$
b. $62.1\ \text{mph}$

Exercise:

Problem:

A car is traveling at a speed of $33\ \text{m/s}$. (a) What is its speed in kilometers per hour? (b) Is it exceeding the $90\ \text{km/h}$ speed limit?

Exercise:

Problem:

Show that $1.0\ \text{m/s} = 3.6\ \text{km/h}$. Hint: Show the explicit steps involved in converting $1.0\ \text{m/s} = 3.6\ \text{km/h}$.

Solution:

$$\frac{1.0\ \text{m}}{\text{s}} = \frac{1.0\ \text{m}}{\text{s}} \times \frac{3600\ \text{s}}{1\ \text{hr}} \times \frac{1\ \text{km}}{1000\ \text{m}}$$

$$= 3.6\ \text{km/h}.$$

Exercise:

Problem:

American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

Exercise:

Problem:

Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

Solution:

length: $377\ \text{ft}$; $4.53 \times 10^3\ \text{in.}$ width: $280\ \text{ft}$; $3.3 \times 10^3\ \text{in.}$

Exercise:

Problem:

What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

Exercise:

Problem:

Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

Solution:

8.847 km

Exercise:

Problem: The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

Exercise:

Problem:

Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

Solution:

(a) $1.3 \times 10^{-9}\text{ m}$

(b) 40 km/My

Exercise:

Problem:

(a) Refer to [link] to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

Glossary

physical quantity
: a characteristic or property of an object that can be measured or calculated from other measurements

units
: a standard used for expressing and comparing measurements

SI units
: the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

English units
: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
: units that can only be expressed relative to the procedure used to measure them

derived units
units that can be calculated using algebraic combinations of the fundamental units

second
the SI unit for time, abbreviated (s)

meter
the SI unit for length, abbreviated (m)

kilogram
the SI unit for mass, abbreviated (kg)

metric system
a system in which values can be calculated in factors of 10

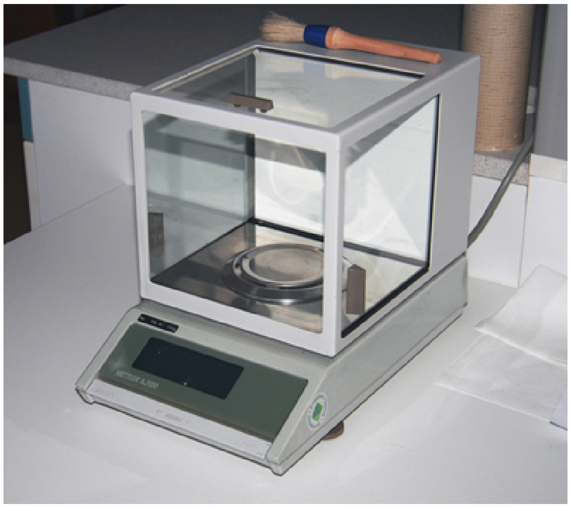
order of magnitude
refers to the size of a quantity as it relates to a power of 10

conversion factor
a ratio expressing how many of one unit are equal to another unit

Accuracy, Precision, and Significant Figures

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)



Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

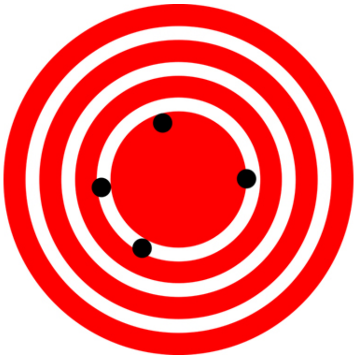
Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in.

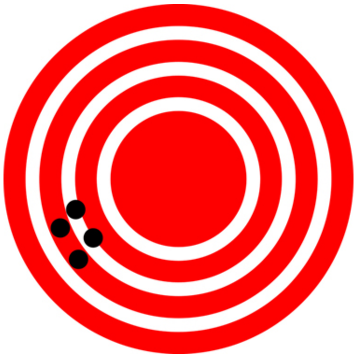
These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull’s-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In [link], you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in [link], the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.



A GPS system attempts to locate a restaurant at the center of the bull’s-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)



In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A "), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2$.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Note:
Making Connections: Real-World Connections – Fevers or Chills?
Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a

hypothermic 34.0^oC to a dangerously high 40.0^oC. A thermometer with an uncertainty of 3.0^oC would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be

Equation:

$$\%\text{ unc} = \frac{\delta A}{A} \times 100\%.$$

Example:

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

Week 1 weight: 4.8 lb
Week 2 weight: 5.3 lb
Week 3 weight: 4.9 lb
Week 4 weight: 5.4 lb

You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\text{ lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

Equation:

$$\% \text{ unc} = \frac{\delta A}{A} \times 100\%.$$

Solution
Plug the known values into the equation:
Equation:

$$\% \text{ unc} = \frac{0.4 \text{ lb}}{5 \text{ lb}} \times 100\% = 8\%.$$

Discussion
We can conclude that the weight of the apple bag is $5 \text{ lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)
Exercise:
Check Your Understanding

Problem:

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ± 0.05 s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Solution:

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Exercise:

Check Your Understanding

Problem:

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Solution:

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\ \text{m}$. Then,

Equation:

$$A = \pi r^2 = (3.1415927...) \times (1.2\ \text{m})^2 = 4.5238934\ \text{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated

quantity to two significant figures or
Equation:

$$A=4.5\ \text{m}^2,$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:
Equation:

$$\begin{array}{rl} & 7.56\ \ \ \text{kg} \\ - & 6.052\ \text{kg} \\ & \frac{+13.7\ \ \ \text{kg}}{15.208\ \text{kg}} = 15.2\ \text{kg}. \end{array}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Exercise:

Check Your Understanding

Problem:

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of $0.0255\ \mathrm{m/s^2}$, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Solution:

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

Note:

PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

https://phet.colorado.edu/sims/estimation/estimation_en.html

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

Conceptual Questions

Exercise:

Problem:

What is the relationship between the accuracy and uncertainty of a measurement?

Exercise:

Problem:

Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

Express your answers to problems in this section to the correct number of significant figures and proper units.

Exercise:

Problem:

Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

Solution:

2 kg

Exercise:

Problem:

A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

Exercise:

Problem:

(a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. $(1\text{ km}=0.6214\text{ mi})$

Solution:

a. $85.5\text{ to }94.5\text{ km/h}$
b. $53.1\text{ to }58.7\text{ mi/h}$

Exercise:

Problem:

An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

Exercise:

Problem:

(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

Solution:

(a) 7.6×10^7 beats

(b) 7.57×10^7 beats

(c) 7.57×10^7 beats

Exercise:

Problem:

A can contains 375 mL of soda. How much is left after 308 mL is removed?

Exercise:

Problem:

State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2)/(46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60 \times 10^{-19}\right)(3712)$.

Solution:

a. 3
b. 3
c. 3

Exercise:

Problem:

(a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

Exercise:

Problem:

(a) If your speedometer has an uncertainty of $2.0\ \mathrm{km/h}$ at a speed of $90\ \mathrm{km/h}$, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads $60\ \mathrm{km/h}$, what is the range of speeds you could be going?

Solution:

a) 2.2%

(b) $59\ \text{to}\ 61\ \mathrm{km/h}$

Exercise:

Problem:

(a) A person's blood pressure is measured to be $120 \pm 2\ \mathrm{mm\ Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of $80\ \mathrm{mm\ Hg}$?

Exercise:

Problem:

A person measures his or her heart rate by counting the number of beats in $30\ \mathrm{s}$. If 40 ± 1 beats are counted in $30.0 \pm 0.5\ \mathrm{s}$, what is the heart rate and its uncertainty in beats per minute?

Solution:

$80 \pm 3 \text{ beats/min}$

Exercise:

Problem: What is the area of a circle 3.102 cm in diameter?

Exercise:

Problem:

If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

Solution:

2.8 h

Exercise:

Problem:

A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

Exercise:

Problem:

The sides of a small rectangular box are measured to be $1.80 \pm 0.01 \text{ cm}$, $2.05 \pm 0.02 \text{ cm}$, and $3.1 \pm 0.1 \text{ cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

Solution:

$11 \pm 1 \text{ cm}^3$

Exercise:

Problem:

When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1\ \text{lbm} = 0.4539\ \text{kg}$. (a) If there is an uncertainty of $0.0001\ \text{kg}$ in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

Exercise:

Problem:

The length and width of a rectangular room are measured to be $3.955 \pm 0.005\ \text{m}$ and $3.050 \pm 0.005\ \text{m}$. Calculate the area of the room and its uncertainty in square meters.

Solution:

$12.06 \pm 0.04\ \text{m}^2$

Exercise:

Problem:

A car engine moves a piston with a circular cross section of $7.500 \pm 0.002\ \text{cm}$ diameter a distance of $3.250 \pm 0.001\ \text{cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

Glossary

accuracy
: the degree to which a measured value agrees with correct value for that measurement

method of adding percents

the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

percent uncertainty
the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

precision
the degree to which repeated measurements agree with each other

significant figures
express the precision of a measuring tool used to measure a value

uncertainty
a quantitative measure of how much your measured values deviate from a standard or expected value

Approximation

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Example:

Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

Equation:

$$\frac{2\text{ m}}{1\text{ person}} \times \frac{2\text{ person}}{1\text{ story}} \times 39\text{ stories} = 156\text{ m}.$$

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Example:

Approximating Vast Numbers: a Trillion Dollars

A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

Equation:

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height},\\ \text{volume of stack} &= 6\text{ in.} \times 3\text{ in.} \times 0.5\text{ in.},\\ \text{volume of stack} &= 9\text{ in.}^3.\end{aligned}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $100 bills is equal to $\$10{,}000$, or $\$1 \times 10^4$. The number of stacks you will have is:

Equation:

$$\$1 \times 10^{12}(\text{a trillion dollars})/\ \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.}$$

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{ yd}\times 50\text{ yd}$, which gives $5{,}000\text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches:

Equation:

$$\begin{gathered}\text{Area} = 5{,}000\text{ yd}^2 \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{12\text{ in.}}{1\text{ ft}} \times \frac{12\text{ in.}}{1\text{ ft}} = 6{,}480{,}000\text{ in.}^2,\\ \text{Area} \approx 6 \times 10^6\text{ in.}^2.\end{gathered}$$

This conversion gives us $6 \times 10^6\text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $100-bill stacks is $9\text{ in.}^3/\text{stack} \times 10^8\text{ stacks} = 9 \times 10^8\text{ in.}^3$.
(5) Calculate the height. To determine the height of the bills, use the equation:

Equation:

$$\begin{aligned} \text{volume of bills} &= \text{area of field} \times \text{height of money:} \\ \text{Height of money} &= \tfrac{\text{volume of bills}}{\text{area of field}}, \\ \text{Height of money} &= \tfrac{9\times10^8\text{in.}^3}{6\times10^6\text{in.}^2} = 1.33 \times 10^2\text{in.}, \\ \text{Height of money} &\approx 1 \times 10^2\text{in.} = 100\text{ in.} \end{aligned}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

Equation:

$$100\text{ in.} \times \frac{1\text{ ft}}{12\text{ in.}} = 8.33\text{ ft} \approx 8\text{ ft.}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

Exercise:
Check Your Understanding

Problem:

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Solution:

An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of $420\ \text{m}^2$.

Summary

Scientists often approximate the values of quantities to perform calculations and analyze systems.

Problems & Exercises

Exercise:

Problem: How many heartbeats are there in a lifetime?

Solution:

Sample answer: 2×10^9 heartbeats

Exercise:

Problem:

A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

Exercise:

Problem:

How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of 10^{-22} s.)

Solution:

Sample answer: 2×10^{31} if an average human lifetime is taken to be about 70 years.

Exercise:

Problem:

Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg.)

This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

Exercise:

Problem:

Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

Solution:

Sample answer: 50 atoms

Exercise:

Problem:

(a) What fraction of Earth’s diameter is the greatest ocean depth? (b) The greatest mountain height?

Exercise:

Problem:

(a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

Solution:

Sample answers:

(a) 10^{12} cells/hummingbird

(b) 10^{16} cells/human

Exercise:

Problem:

Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

Glossary

approximation

an estimated value based on prior experience and reasoning

Introduction to One-Dimensional Kinematics

class="introduction"

https://www.openstaxcollege.org/l/9nmv4Q

The motion of an American kestrel through the air can be described by the bird’s displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

https://www.openstaxcollege.org/l/9nmv4Q

The motion of an American kestrel through the air can be described by the bird’s displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

https://www.openstaxcollege.org/l/9nmv4Q

The motion of an American kestrel through the air can be described by the bird’s displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See [link].) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See [link].)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

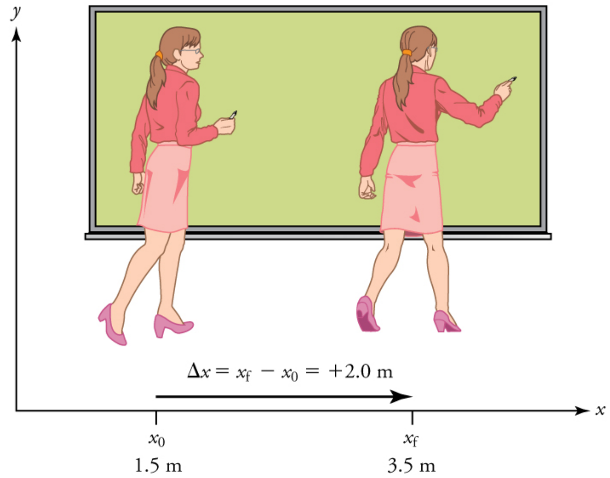
Note:
Displacement
Displacement is the *change in position* of an object:
Equation:

$$\Delta x = x_{\text{f}} - x_0,$$

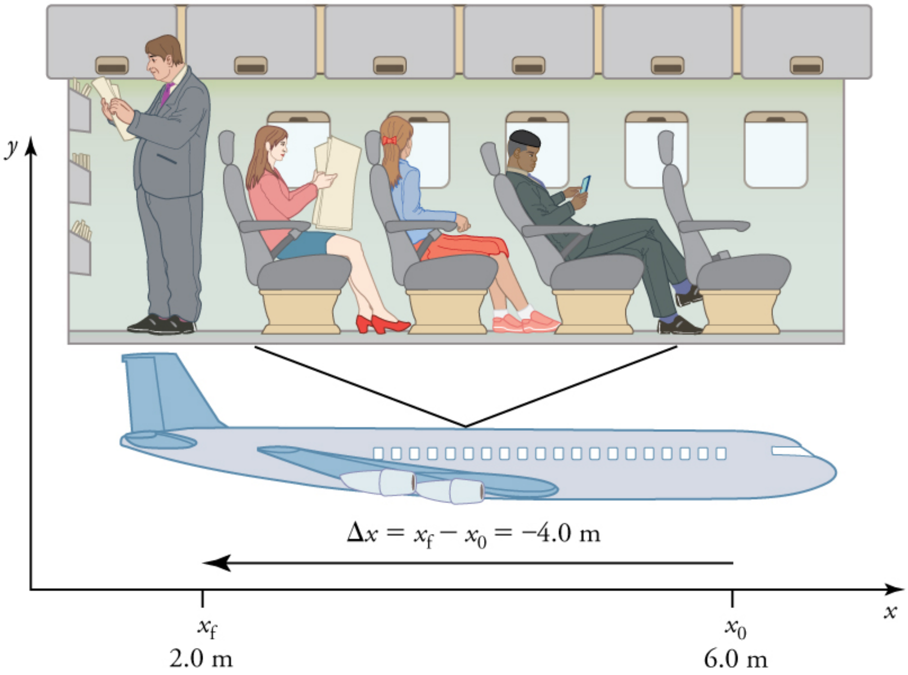
where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_{f}.

Note that the SI unit for displacement is the meter (m) (see Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.


y
$\Delta x = x_f - x_0 = +2.0$ m
x
x_0
1.5 m
x_f
3.5 m

A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0$ m displacement of the professor relative to Earth is represented by an arrow pointing to the right.



y
$\Delta x = x_f - x_0 = -4.0$ m
x
x_f
2.0 m
x_0
6.0 m

A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in [link].

Note that displacement has a direction as well as a magnitude. The professor’s displacement is 2.0 m to the right, and the airline passenger’s displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor’s initial position is $x_0 = 1.5 \text{ m}$ and her final position is $x_\text{f} = 3.5 \text{ m}$. Thus her displacement is

Equation:

$$\Delta x = x_\text{f} - x_0 = 3.5 \text{ m} - 1.5 \text{ m} = +2.0 \text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{ m}$ and his final position is $x_\text{f} = 2.0\text{ m}$, so his displacement is
Equation:

$$\Delta x = x_\text{f} - x_0 = 2.0\text{ m} - 6.0\text{ m} = -4.0\text{ m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Note:
Misconception Alert: Distance Traveled vs. Magnitude of Displacement
It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The

displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

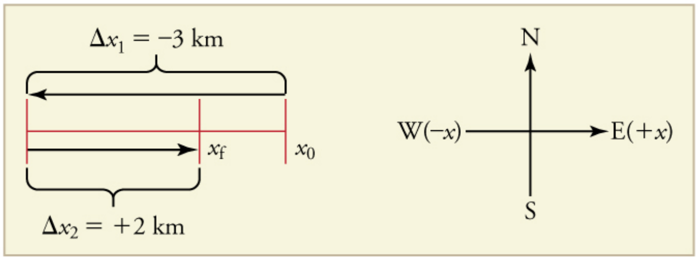
Exercise:

Check Your Understanding

Problem:

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution:



(a) The rider’s displacement is $\Delta x = x_{\text{f}} - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km.

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

- In symbols, displacement Δx is defined to be
  Equation:

  $$\Delta x = x_{\text{f}} - x_0,$$

  where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means “change in” whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Conceptual Questions

Exercise:

Problem:

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

Exercise:

Problem:

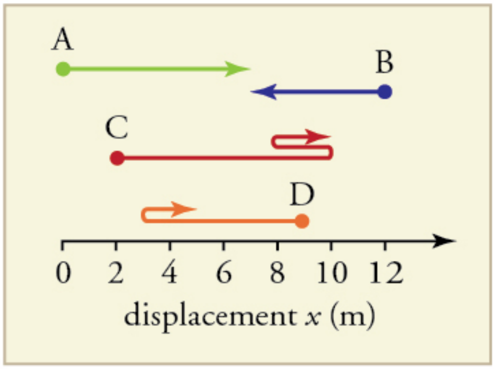
Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

Exercise:

Problem:

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}$ $\left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

Problems & Exercises



Exercise:

Problem:

Find the following for path A in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

(a) 7 m

(b) 7 m

(c) $+7$ m

Exercise:

Problem:

Find the following for path B in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Exercise:

Problem:

Find the following for path C in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

(a) 13 m

(b) 9 m

(c) +9 m

Exercise:

Problem:

Find the following for path D in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Glossary

kinematics	the study of motion without considering its causes
position	the location of an object at a particular time
displacement	the change in position of an object
distance	the magnitude of displacement between two positions
distance traveled	the total length of the path traveled between two positions

Vectors, Scalars, and Coordinate Systems

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

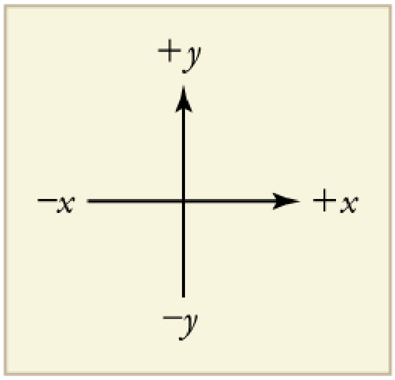
The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Note:
Video Explorations: Vectors & Scalars
This video introduces and differentiates between vectors and scalars. It also introduces quantities that we will be working with during the study of kinematics.
https://openstaxcollege.org/l/28vectorscalar

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in [link], it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.



It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(−)$.

Exercise:

Check Your Understanding

Problem:

A person’s speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution:

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Conceptual Questions

Exercise:

Problem:

A student writes, “*A bird that is diving for prey has a speed of* $-10\ m/s$.” What is wrong with the student’s statement? What has the student actually described? Explain.

Exercise:

Problem: What is the speed of the bird in [link]?

Exercise:

Problem:

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

Exercise:

Problem:

A weather forecast states that the temperature is predicted to be -5°C the following day. Is this temperature a vector or a scalar quantity? Explain.

Glossary

scalar
a quantity that is described by magnitude, but not direction

vector
a quantity that is described by both magnitude and direction

Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

Equation:

$$\Delta t = t_\mathrm{f} - t_0,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_\mathrm{f} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_\mathrm{f} \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Note:
Average Velocity
Average velocity is *displacement (change in position) divided by the time of travel*,
Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_{f} and x_0 are the final and beginning positions at times t_{f} and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply
Equation:

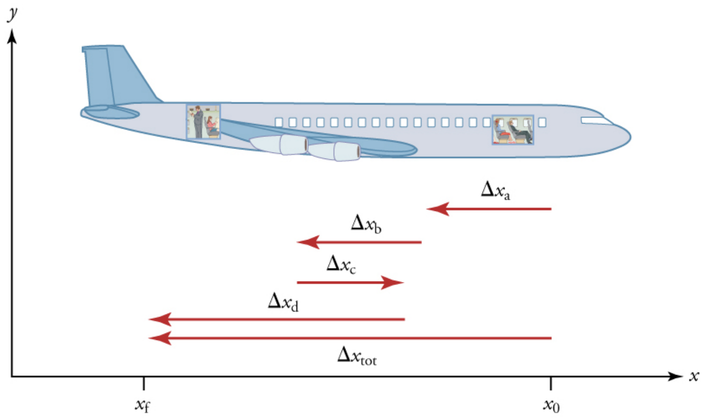
$$\bar{v} = \frac{\Delta x}{t}.$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be
Equation:

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\ \text{m}}{5\ \text{s}} = -0.8\ \text{m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms “speed” and “velocity” interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

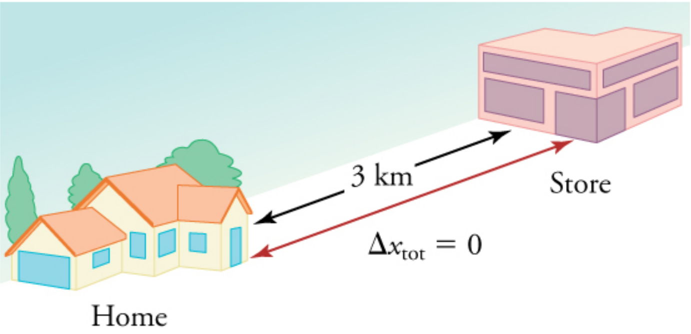
Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

Note:
PhET Explorations: The Walking Man
In this simulation you will put your cursor on the man and move him first in one direction and then in the opposite direction. Keep the “Introduction” tab active. You can use the “Charts” tab after you learn about graphing

motion later in this chapter. Carefully watch the sign of the numbers in the position and velocity boxes. Ignore the acceleration box for now. See if you can make the man’s position positive while the velocity is negative. Then see if you can do the opposite.

Click here for the simulation

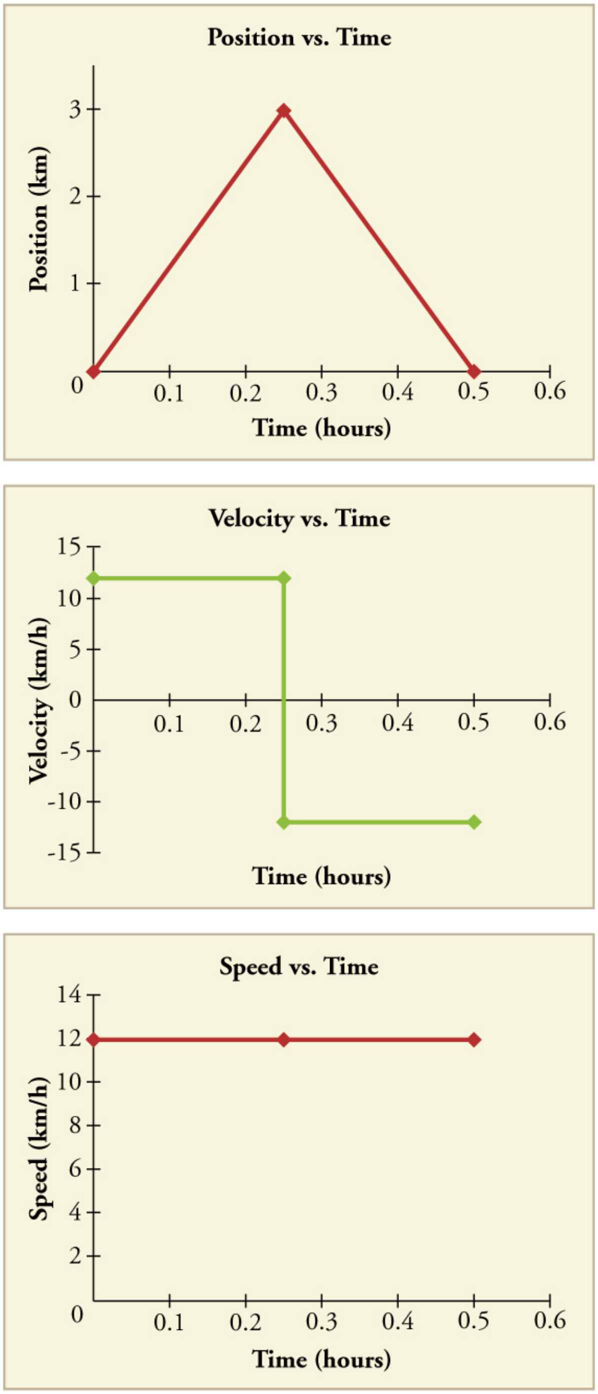
We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car’s odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.



During a 30-minute round trip to the

store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in [link]. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we’ll probably stop at the store. But for simplicity’s sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)



Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Note:

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Exercise:

Check Your Understanding

Problem:

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution:

(a) The average velocity of the train is zero because $x_\mathrm{f} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

Equation:

$$\frac{\text{distance}}{\text{time}} = \frac{80\text{ miles}}{105\text{ minutes}}$$

Equation:

$$\frac{80\text{ miles}}{105\text{ minutes}} \times \frac{5280\text{ feet}}{1\text{ mile}} \times \frac{1\text{ meter}}{3.28\text{ feet}} \times \frac{1\text{ minute}}{60\text{ seconds}} = 20\text{ m/s}$$

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  Equation:
  $$\Delta t = t_\text{f} - t_0,$$
  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is
  Equation:
  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0}.$$
- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Conceptual Questions

Exercise:

Problem:

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

Exercise:

Problem:

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

Exercise:

Problem:

Does a car’s odometer measure position or displacement? Does its speedometer measure speed or velocity?

Exercise:

Problem:

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

Exercise:

Problem:

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

Problems & Exercises

Exercise:

Problem:

(a) Calculate Earth’s average speed relative to the Sun. (b) What is its average velocity over a period of one year?

Solution:

(a) $3.0 \times 10^4 \text{ m/s}$

(b) 0 m/s

Exercise:

Problem:

A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter’s frame of reference. (b) What is its average velocity over one revolution?

Exercise:

Problem:

The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

Solution:

2×10^7 years

Exercise:

Problem:

Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

Exercise:

Problem:

On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

Solution:

$34.689\text{ m/s}=124.88\text{ km/h}$

Exercise:

Problem:

Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84\times10^{6}\text{ m}$ (1%)?

Exercise:

Problem:

A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

Solution:

(a) $40.0\ \mathrm{km/h}$

(b) 34.3 km/h, $25^{\circ}\ \mathrm{S\ of\ E.}$

(c) $\text{average speed} = 3.20\ \mathrm{km/h},\ \bar{v} = 0.$

Exercise:

Problem:

The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

Exercise:

Problem:

Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person’s voice was so loud in the astronaut’s space helmet that it was picked up by the astronaut’s microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00 \times 10^8 \text{ m/s})$.

Solution:

384,000 km

Exercise:

Problem:

A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

Exercise:

Problem:

The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10} \text{ m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6 \text{ m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron’s average velocity?

Solution:

(a) $6.61 \times 10^{15}\ \mathrm{rev/s}$

(b) 0 m/s

Glossary

average speed
distance traveled divided by time during which motion occurs

average velocity
displacement divided by time over which displacement occurs

instantaneous velocity
velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
magnitude of the instantaneous velocity

time
change, or the interval over which change occurs

model
simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
the difference between the ending time and beginning time

Acceleration

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Note:
Average Acceleration
Average Acceleration is *the rate at which velocity changes,*
Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Note:

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

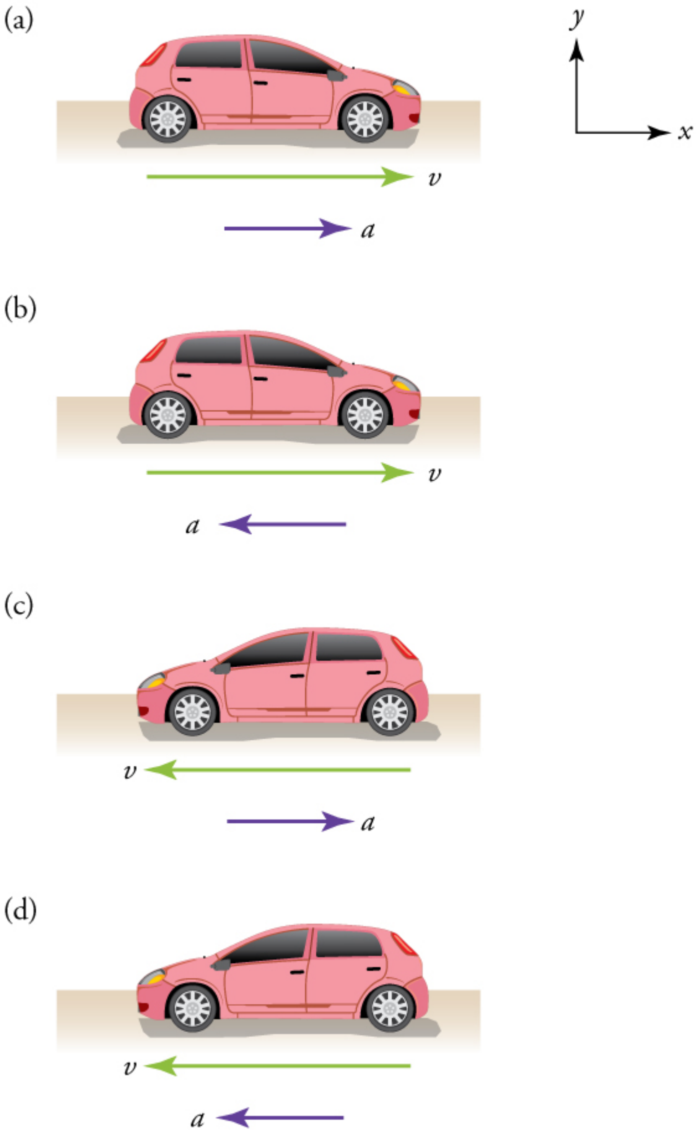
Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Note:

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider [link].



(a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

Example:
Calculating Projectile Motion: A Baseball LINK ONLY

A baseball is thrown straight up with an initial velocity of 15m/s.

- At what time is the baseball's velocity 0m/s?
- What is the ball's height at that time?
- Make a displacement-time graph of the ball's motion, labeling any points that you know.
- Make a velocity-time graph of the ball's motion, labeling any points that you know.

Study the problem and its solution in this video.

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- Make a velocity-time graph of the ball's motion, labeling any points that you know.

Study the problem and its solution in this video: https://www.openstaxcollege.org/l/cw_college_physics_sample2

Example:

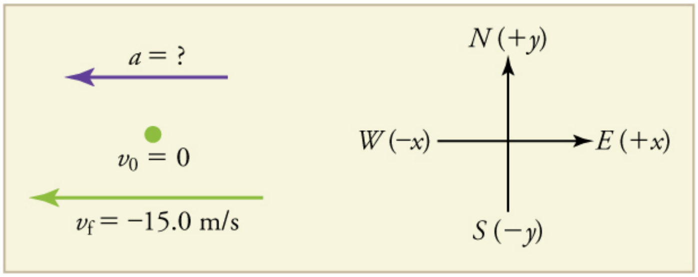
Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

(credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\ \text{m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\ \text{s}$.

2. Find the change in velocity. Since the horse is going from zero to $-15.0\ \text{m/s}$, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\ \text{m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

Equation:

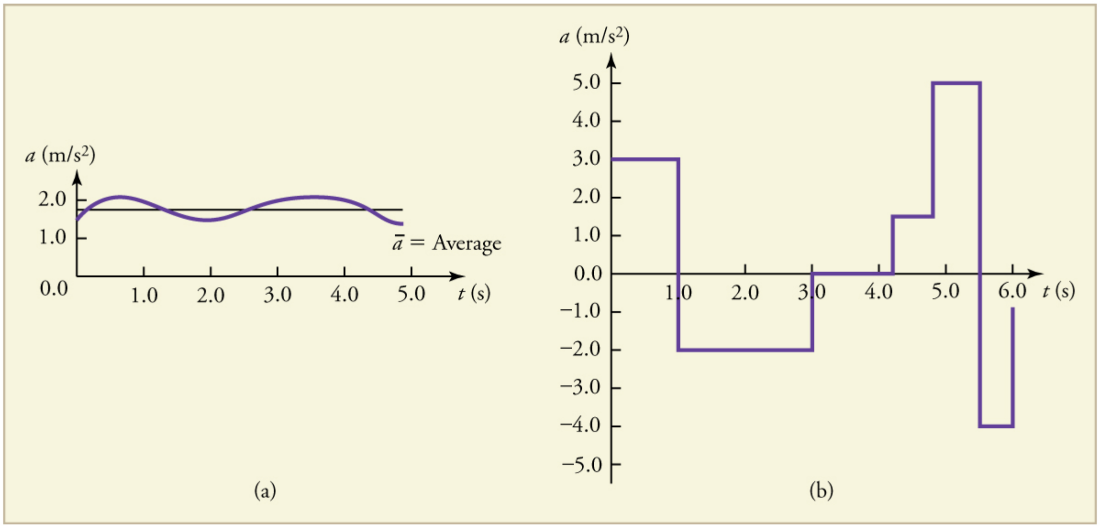
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\ \text{m/s}}{1.80\ \text{s}} = -8.33\ \text{m/s}^2.$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of $8.33\ \text{m/s}^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as $8.33\ \text{m/s}^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

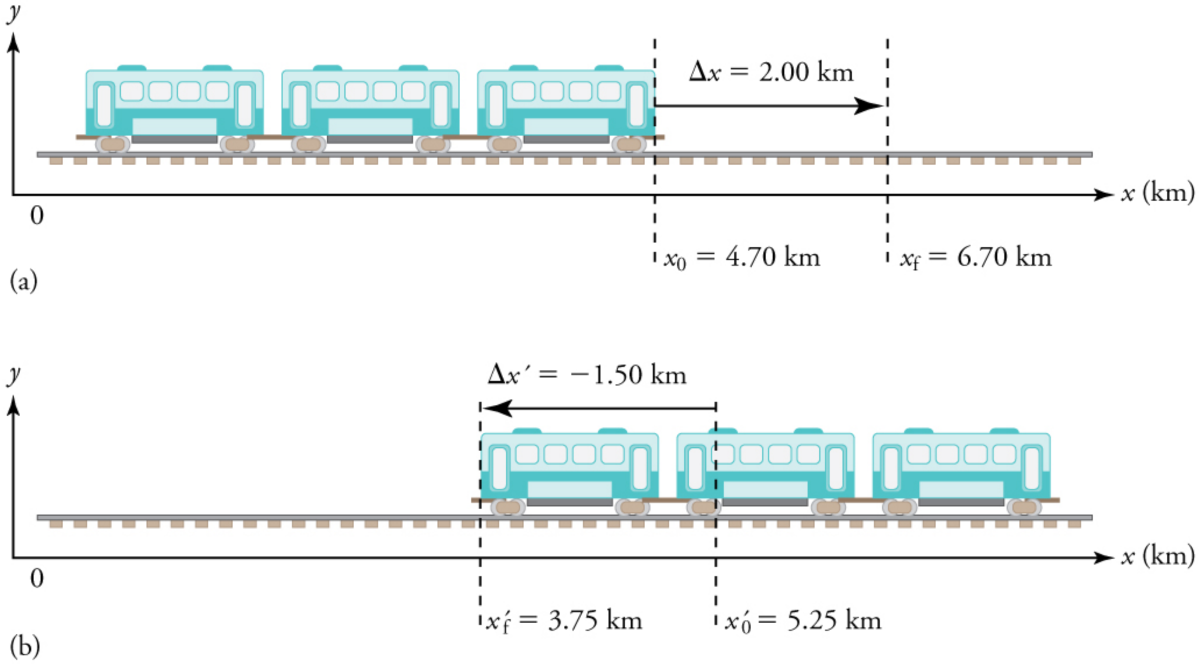
Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. [link] shows graphs of instantaneous acceleration versus time for two very different motions. In [link](a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In [link](b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and $–2.0\text{ m/s}^2$, respectively.



Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in [link]. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.



One-dimensional motion of a subway train considered in [link], [link], [link], [link], [link], and [link]. Here we have chosen the x-axis so that + means to the right and $-$ means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from $x\prime_0$ to $x\prime_f$. Its displacement $\Delta x\prime$ is -1.5 km. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example:

Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of [link]?

Strategy

A drawing with a coordinate system is already provided, so we don’t need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_{\text{f}} - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_{\text{f}} = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x'_{\text{f}} = 3.75\text{ km}$ and $x'_0 = 5.25\text{ km}$ for part (b).

2. Solve for displacement in part (a).

Equation:

$$\Delta x = x_{\text{f}} - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km}$$

3. Solve for displacement in part (b).

Equation:

$$\Delta x' = x'_{\text{f}} - x'_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example:

Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in [link]?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in [link]. Distance traveled is the total length of the path traveled between the two positions. (See Displacement.) In the case of the subway train shown in [link], the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.
2. The displacement for part (b) was -1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

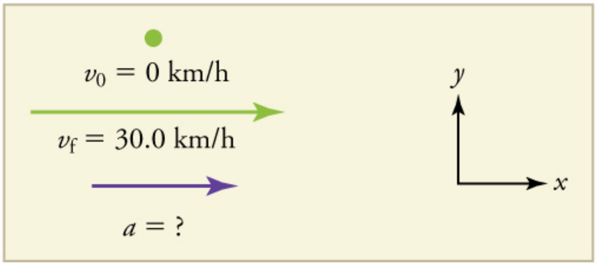
Example:

Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in [link](a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:



This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0\text{ km/h}$, and $\Delta t = 20.0\text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\text{ km/h}}{20.0\text{ s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

Equation:

$$\bar{a} = \left(\frac{+30\text{ km/h}}{20.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.417\text{ m/s}^2$$

Discussion

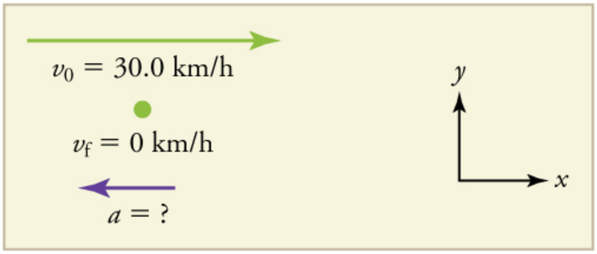
The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example:

Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in [link](a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy



In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\text{ km/h}$, $v_\text{f} = 0\text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\text{ s}$.

2. Solve for the change in velocity, Δv.

Equation:

$$\Delta v = v_\text{f} - v_0 = 0 - 30.0\text{ km/h} = -30.0\text{ km/h}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\text{ km/h}}{8.00\text{ s}}$$

4. Convert the units to meters and seconds.

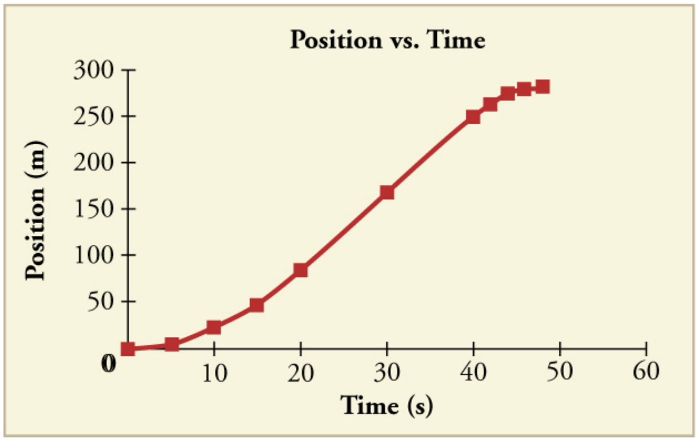
Equation:

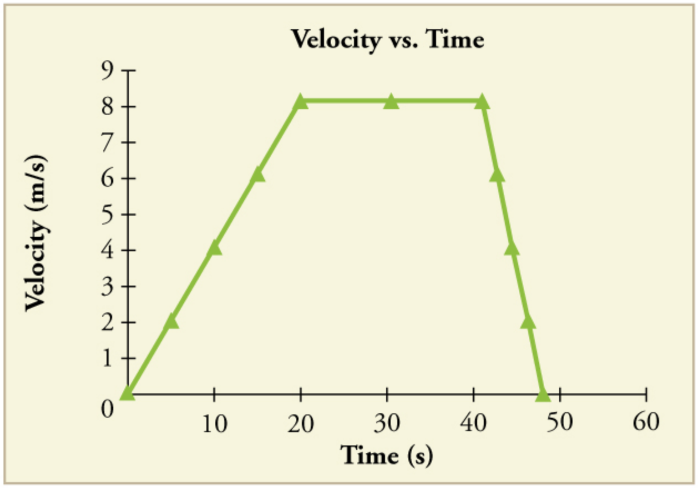
$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\text{ km/h}}{8.00\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -1.04\text{ m/s}^2.$$

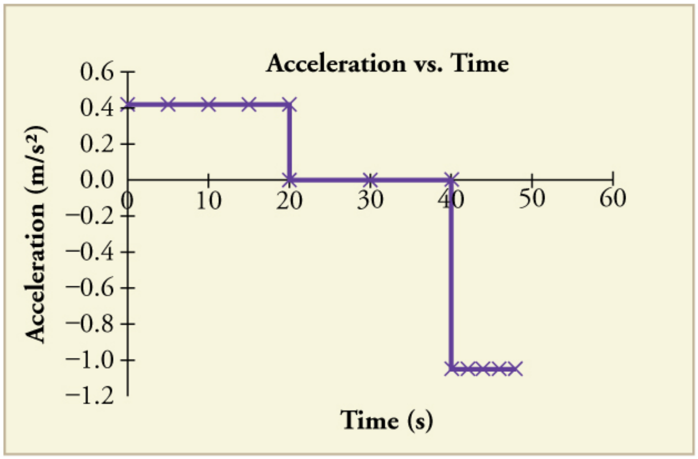
Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in [link] and [link] are displayed in [link]. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)





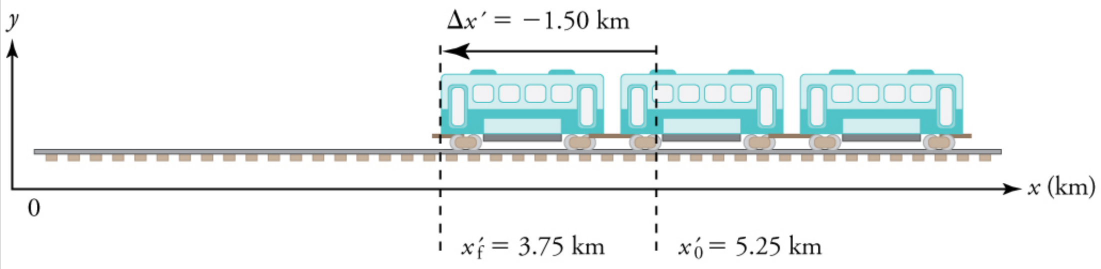


(a) Position of the train over time. Notice that the train’s position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train’s velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example:

Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of [link], and shown again below, if it takes 5.00 min to make its trip?



Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x\prime_{\text{f}} = 3.75\text{ km}$, $x\prime_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.
2. Determine displacement, $\Delta x\prime$. We found $\Delta x\prime$ to be -1.5 km in [link].
3. Solve for average velocity.

Equation:

$$\bar{v} = \frac{\Delta x\prime}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}}$$

4. Convert units.

Equation:

$$\bar{v} = \frac{\Delta x\prime}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h}$$

Discussion

The negative velocity indicates motion to the left.

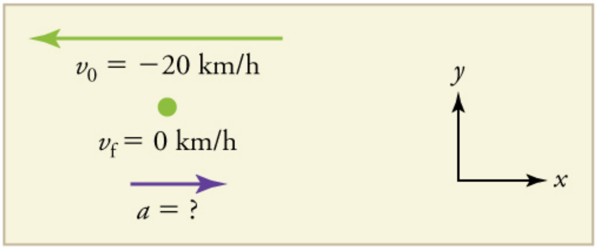
Example:

Calculating Deceleration: The Subway Train

Finally, suppose the train in [link] slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let’s draw a sketch:



As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20 \text{ km/h}$, $v_f = 0 \text{ km/h}$, $\Delta t = 10.0 \text{ s}$.

2. Calculate Δv. The change in velocity here is actually positive, since

Equation:

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \text{ s}}$$

4. Convert units.

Equation:

$$\bar{a} = \left(\frac{+20.0 \text{ km/h}}{10.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = +0.556 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in [link], this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in [link], where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in [link] is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Exercise:

Check Your Understanding

Problem:

An airplane lands on a runway traveling east. Describe its acceleration.

Solution:

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

Note:

PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Moving Man

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is
  Equation:

  $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\mathrm{f} - v_0}{t_\mathrm{f} - t_0}.$$
- The SI unit for acceleration is $\mathrm{m/s}^2$.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Conceptual Questions

Exercise:

Problem:

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

Exercise:

Problem:

Is it possible for velocity to be constant while acceleration is not zero? Explain.

Exercise:

Problem:

Give an example in which velocity is zero yet acceleration is not.

Exercise:

Problem:

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

Exercise:

Problem:

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

Problems & Exercises

Exercise:

Problem:

A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

Solution:

4.29 m/s^2

Exercise:

Problem: **Professional Application**

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g $(9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

Exercise:

Problem:

A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

Solution:

(a) $1.43\ \text{s}$

(b) $-2.50\ \text{m/s}^2$

Exercise:

Problem:

Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g $(9.80\ \text{m/s}^2)$?

Glossary

acceleration	the rate of change in velocity; the change in velocity over time
average acceleration	the change in velocity divided by the time over which it changes
instantaneous acceleration	

acceleration at a specific point in time

deceleration
acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_{\mathrm{f}} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_{\mathrm{f}}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

Equation:

$$\begin{array}{rcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

Equation:

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Note:
Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields

Equation:

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for x yields

Equation:

$$x = x_0 + \bar{v}t,$$

where the average velocity is

Equation:

$$\bar{v} = \frac{v_0 + v}{2} \ (\text{constant } a).$$

The equation $\bar{v} = \frac{v_0+v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0+v}{2}$ to check this, we see that

Equation:

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h},$$

which seems logical.

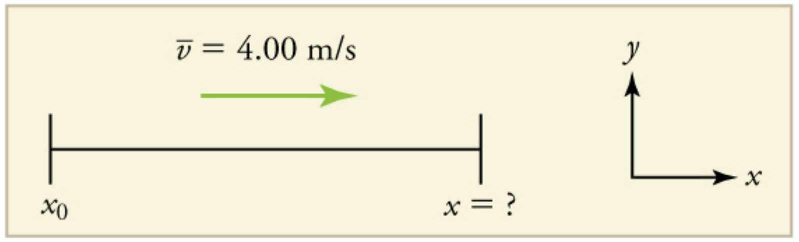
Example:

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.



The final position x is given by the equation

Equation:

$$x = x_0 + \bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.
2. Enter the known values into the equation.

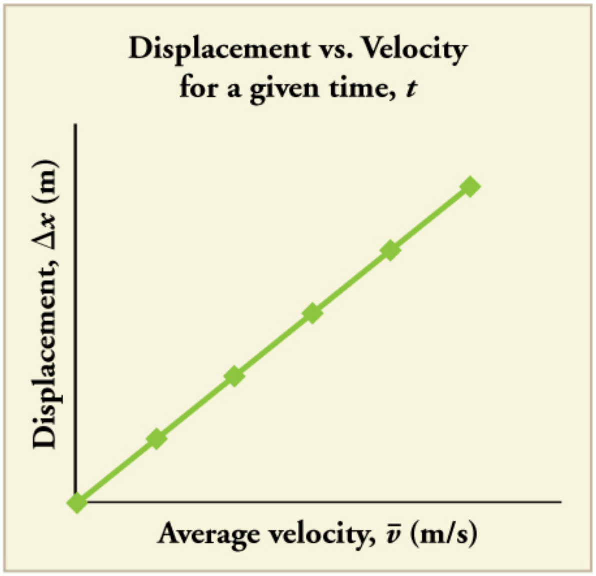
Equation:

$$x = x_0 + \bar{v}t = 0 + (4.00\text{ m/s})(120\text{ s}) = 480\text{ m}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will

move twice as far as the other
object.

Note:

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

Equation:

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

Equation:

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{).}$$

Solving for v yields

Equation:

$$v = v_0 + \text{at} \text{ (constant } a\text{).}$$

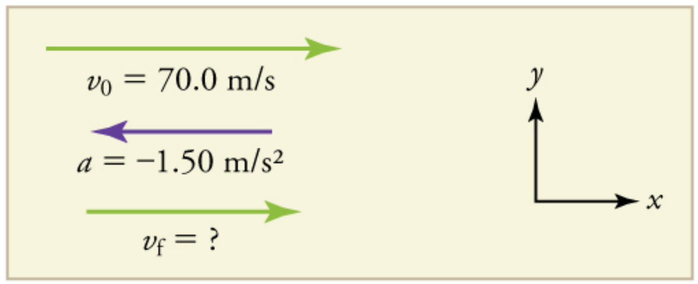
Example:

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.



Solution

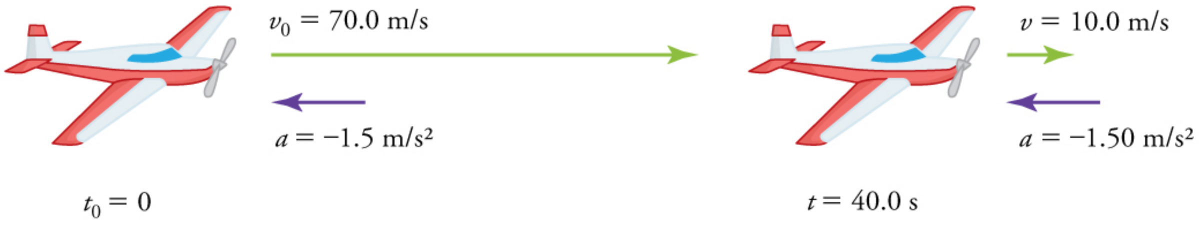
1. Identify the knowns. $v_0 = 70.0\ \mathrm{m/s}$, $a = -1.50\ \mathrm{m/s}^2$, $t = 40.0\ \mathrm{s}$.
2. Identify the unknown. In this case, it is final velocity, v_{f}.
3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + \mathrm{at}$.
4. Plug in the known values and solve.

Equation:

$$v = v_0 + \mathrm{at} = 70.0\ \mathrm{m/s} + \left(-1.50\ \mathrm{m/s}^2\right)(40.0\ \mathrm{s}) = 10.0\ \mathrm{m/s}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + \text{at}$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Note:
Making Connections: Real-World Connection

The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Note:

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

Equation:

$$v = v_0 + \text{at}.$$

Adding v_0 to each side of this equation and dividing by 2 gives

Equation:

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}\text{at}.$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, then

Equation:

$$\bar{v} = v_0 + \frac{1}{2}\text{at}.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}.$$

Example:

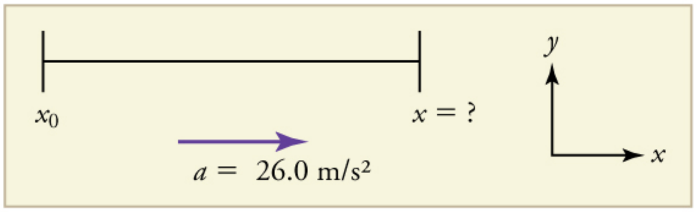
Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of $26.0\ \text{m/s}^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.



We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0\ \text{m/s}^2$ and t is given as 5.56 s.
2. Plug the known values into the equation to solve for the unknown x:

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Since the initial position and velocity are both zero, this simplifies to

Equation:

$$x = \frac{1}{2}at^2.$$

Substituting the identified values of a and t gives

Equation:

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)(5.56\ \text{s})^2,$$

yielding

Equation:

$$x = 402\ \text{m}.$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In [link], the dragster covers only one fourth of the total distance in the first half of the elapsed time

- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Note:

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 +$ at for t, we get

Equation:

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

Equation:

$$v^2 = v_0^2 + 2a(x - x_0)\ (\text{constant } a).$$

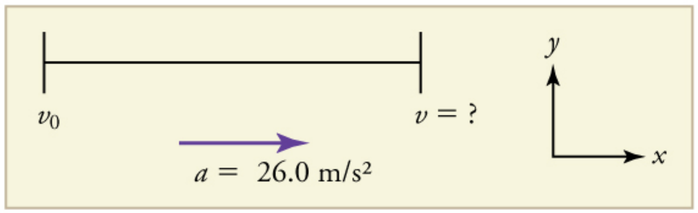
Example:

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in [link] without using information about time.

Strategy

Draw a sketch.



The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\text{ m}$ (this was the answer in [link]). Finally, the average acceleration was given to be $a = 26.0\text{ m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

Equation:

$$v^2 = 0 + 2\left(26.0\text{ m/s}^2\right)(402\text{ m}).$$

Thus

Equation:

$$v^2 = 2.09 \times 10^4\text{ m}^2/\text{s}^2.$$

To get v, we take the square root:

Equation:

$$v = \sqrt{2.09 \times 10^4\text{ m}^2/\text{s}^2} = 145\text{ m/s}.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why

we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Note:
Summary of Kinematic Equations (constant a)
Equation:

$$x = x_0 + \bar{v}t$$

Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

Equation:

$$v = v_0 + \text{at}$$

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

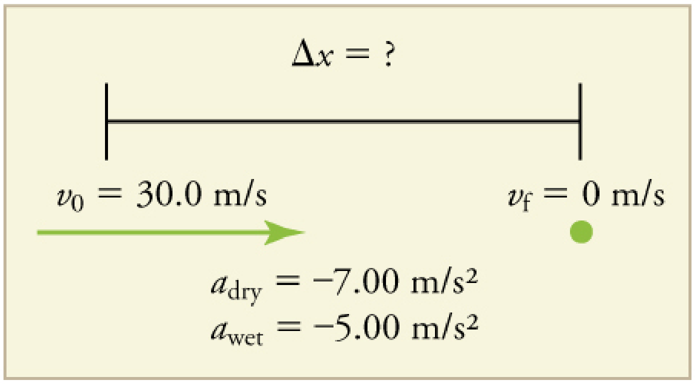
Example:

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00\ \mathrm{m/s}^2$, whereas on wet concrete it can decelerate at only $5.00\ \mathrm{m/s}^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \mathrm{m/s}$; $v = 0$; $a = -7.00\ \mathrm{m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

Equation:

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

Equation:

$$x - 0 = \frac{0^2 - (30.0\text{ m/s})^2}{2\left(-7.00\text{ m/s}^2\right)}$$

Thus,

Equation:

$$x = 64.3\text{ m on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2. The result is

Equation:

$$x_{\text{wet}} = 90.0\text{ m on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver’s reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\text{ m/s}$; $t_{\text{reaction}} = 0.500\,\text{s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.
3. Plug in the knowns to solve the equation.

Equation:

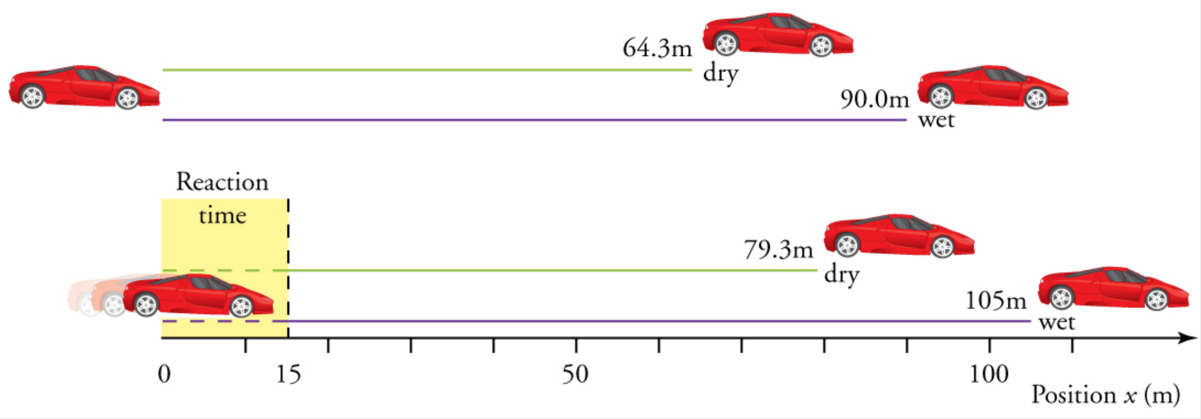
$$x = 0 + (30.0\text{ m/s})(0.500\text{ s}) = 15.0\text{ m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.
4. Add the displacement during the reaction time to the displacement when braking.

Equation:

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

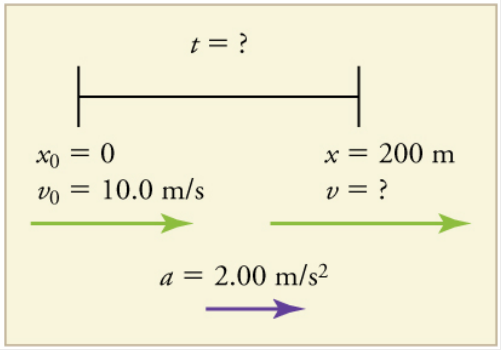
The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example:
Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at $2.00\ \mathrm{m/s}^2$, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.



We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\ \mathrm{m/s}$; $a = 2.00\ \mathrm{m/s}^2$; and $x = 200\ \mathrm{m}$.
2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

Equation:

$$200\ \mathrm{m} = 0\ \mathrm{m} + (10.0\ \mathrm{m/s})t + \frac{1}{2}\left(2.00\ \mathrm{m/s}^2\right)t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t\ \mathrm{s}$, where t is the magnitude of time and s is the unit. Doing so leaves

Equation:

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

Equation:

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

Equation:

$$at^2 + bt + c = 0,$$

where the constants are $a = 1.00$, $b = 10.0$, and $c = -200$.

(b) Its solutions are given by the quadratic formula:

Equation:

$$t = \frac{-b \pm \sqrt{b^2 - 4\mathrm{ac}}}{2a}.$$

This yields two solutions for t, which are

Equation:

$$t = 10.0\ \text{and} - 20.0.$$

In this case, then, the time is $t = t$ in seconds, or

Equation:

$$t = 10.0\text{ s and } - 20.0\text{ s.}$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

Equation:

$$t = 10.0\text{ s.}$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Note:

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Exercise:
Check Your Understanding

Problem:

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Solution:

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.
Equation:

$$v = v_0 + \text{at}$$

Rearrange to solve for t.
Equation:

$$t = \frac{v - v_0}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s}$$

Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,
  Equation:

$$\begin{aligned} \Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0 \end{aligned}$$

- The following kinematic equations for motion with constant a are useful:
  Equation:

  $$x = x_0 + \bar{v}t$$

  Equation:

  $$\bar{v} = \frac{v_0 + v}{2}$$

  Equation:

  $$v = v_0 + \text{at}$$

  Equation:

  $$x = x_0 + v_0 t + \frac{1}{2}at^2$$

  Equation:

  $$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

Problems & Exercises

Exercise:

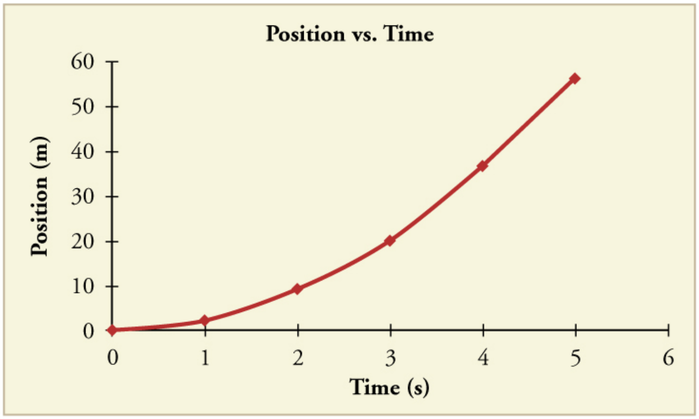
Problem:

An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

Solution:

(a) $10.8\ \text{m/s}$

(b)



Exercise:

Problem:

A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4 \text{ m/s}^2$, and 1.85 ms $(1 \text{ ms} = 10^{-3} \text{ s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

Solution:

38.9 m/s (about 87 miles per hour)

Exercise:

Problem:

A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5 \text{ m/s}^2$ for 8.10×10^{-4} s. What is its muzzle velocity (that is, its final velocity)?

Exercise:

Problem:

(a) A light-rail commuter train accelerates at a rate of $1.35\ \text{m/s}^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of $1.65\ \text{m/s}^2$. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

Solution:

(a) $16.5\ \text{s}$

(b) $13.5\ \text{s}$

(c) $-2.68\ \text{m/s}^2$

Exercise:

Problem:

While entering a freeway, a car accelerates from rest at a rate of $2.40\ \text{m/s}^2$ for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

Exercise:

Problem:

At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00\ \text{m/s}^2$. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

Solution:

(a) 20.0 m

(b) -1.00 m/s

(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 m/s^2, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

Exercise:

Problem:Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

Exercise:

Problem:

In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes 3.33×10^{-2} s, calculate the distance over which the puck accelerates.

Solution:

0.799 m

Exercise:

Problem:

A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

Exercise:

Problem:

Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \text{m/s}^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \text{m/s}^2$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

Solution:

(a) $28.0\ \text{m/s}$

(b) $50.9\ \text{s}$

(c) 7.68 km to accelerate and 713 m to decelerate

Exercise:

Problem:

A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

Exercise:

Problem:

A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \text{m/s}^2$, how far will it travel before becoming airborne? (b) How long does this take?

Solution:

(a) $51.4\ \text{m}$

(b) $17.1\ \text{s}$

Exercise:

Problem: Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in m/s^2 and in multiples of g $\left(g = 9.80\ \text{m/s}^2\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

Exercise:

Problem:

An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

Solution:

(a) $-80.4\ \mathrm{m/s}^2$

(b) $9.33\times10^{-2}\ \mathrm{s}$

Exercise:

Problem:

In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

Exercise:

Problem:

Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

Solution:

(a) $7.7\ \mathrm{m/s}$

(b) $-15\times10^2\ \mathrm{m/s}^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

Exercise:

Problem:

An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150\ \text{m/s}^2$ as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

Exercise:

Problem:

Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in [link] and [link]. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

Solution:

(a) $32.6\ \text{m/s}^2$

(b) $162\ \text{m/s}$

(c) $v > v_{\text{max}}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at $32.6\ \text{m/s}^2$ during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

Exercise:

Problem:

A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500\ \text{m/s}^2$ for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

Exercise:

Problem:

In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

Solution:

104 s

Exercise:

Problem:

(a) A world record was set for the men’s 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt “coasted” across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

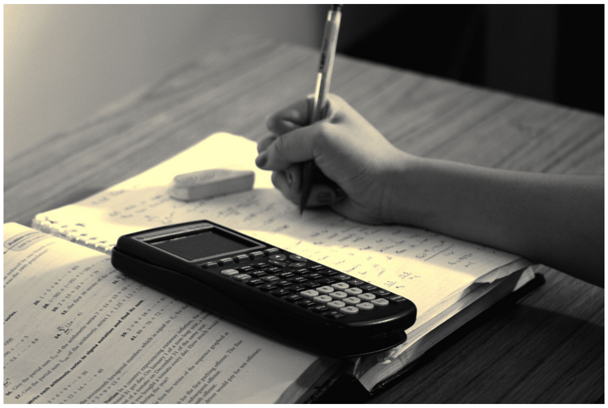
Solution:

(a) $v = 12.2\ \text{m/s}$; $a = 4.07\ \text{m/s}^2$

(b) $v = 11.2\ \text{m/s}$

Problem-Solving Basics for One-Dimensional Kinematics

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.



Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,
Equation:

$$v = v_0 + \text{at} = 0 + \left(0.40\ \text{m/s}^2\right)(100\ \text{s}) = 40\ \text{m/s}.$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

Equation:

$$\left(\frac{40\ \text{m}}{\text{s}}\right)\left(\frac{3.28\ \text{ft}}{\text{m}}\right)\left(\frac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 89\ \text{mph}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at $0.40\ \text{m/s}^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of $0.40\ \text{m/s}^2$ for 100 s (almost two minutes).

Section Summary

- *The six basic problem solving steps for physics are:*

Step 1. Examine the situation to determine which physical principles are involved.

Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Find an equation or set of equations that can help you solve the problem.

Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

Step 6. Check the answer to see if it is reasonable: Does it make sense?

Conceptual Questions

Exercise:

Problem:

What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

Exercise:

Problem:

What is the last thing you should do when solving a problem? Explain.

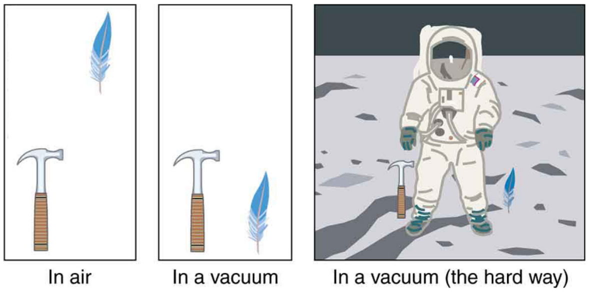
Falling Objects

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.


In air
In a vacuum
In a vacuum (the hard way)

A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the

acceleration due to gravity is
only 1.67 m/s^2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, *g*. It is constant at any given location on Earth and has the average value

Equation:

$$g = 9.80 \text{ m/s}^2.$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80 \text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80 \text{ m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Note:
Kinematic Equations for Objects in Free-Fall where Acceleration = -g
Equation:

$$v = v_0 - \mathrm{gt}$$

Equation:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

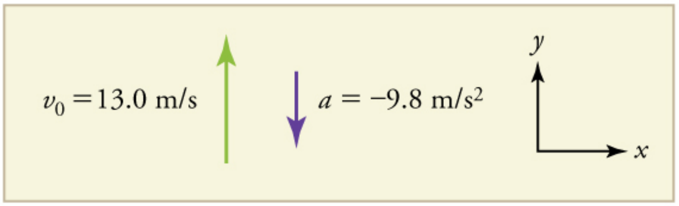
Equation:

$$v^2 = v_0^2 - 2g(y - y_0)$$

Example:
Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward
A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.



We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$; and $t = 1.00 \text{ s}$.
2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

Equation:

$$y_1 = 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(1.00 \text{ s})^2 = 8.10 \text{ m}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$; and $t = 1.00 \text{ s}$. We also know from the solution above that $y_1 = 8.10 \text{ m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - \text{gt}$ (from $v = v_0 + \text{at}$, where $a = \text{gravitational acceleration} = -g$).
3. Plug in the knowns and solve.

Equation:

$$v_1 = v_0 - \text{gt} = 13.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)(1.00\ \text{s}) = 3.20\ \text{m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00$ s. However, it has slowed from its original 13.0 m/s, as expected.

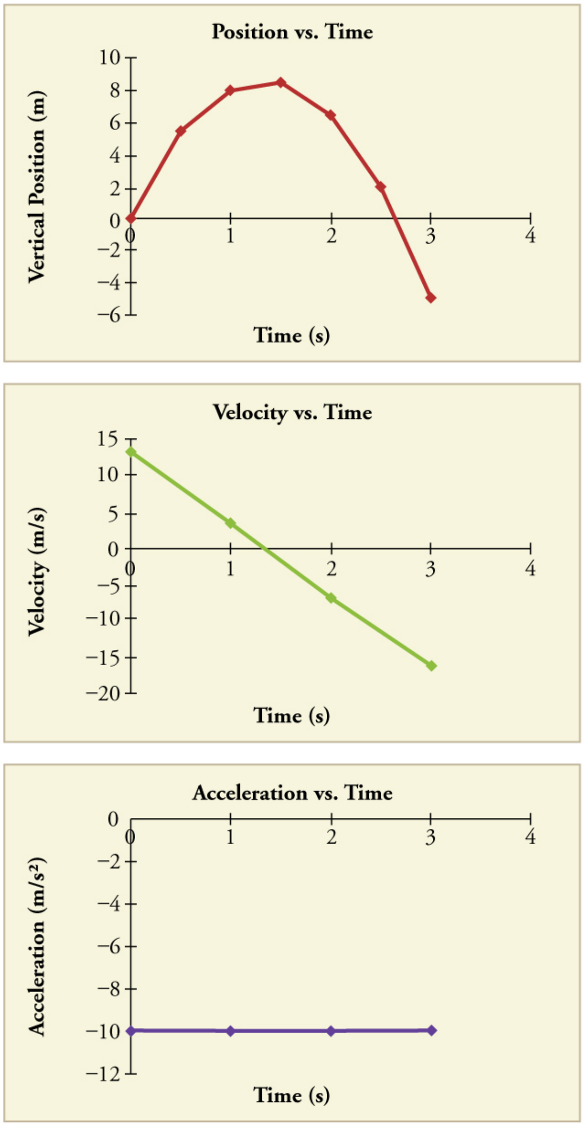
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00$ s and 3.00 s are the same as those above. The results are summarized in [link] and illustrated in [link].

Time, *t*	**Position, *y***	**Velocity, *v***	**Acceleration, *a***
$1.00\ \text{s}$	$8.10\ \text{m}$	$3.20\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$2.00\ \text{s}$	$6.40\ \text{m}$	$-6.60\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$3.00\ \text{s}$	$-5.10\ \text{m}$	$-16.4\ \text{m/s}$	$-9.80\ \text{m/s}^2$

Results

Graphing the data helps us understand it more clearly.



Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Note:

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

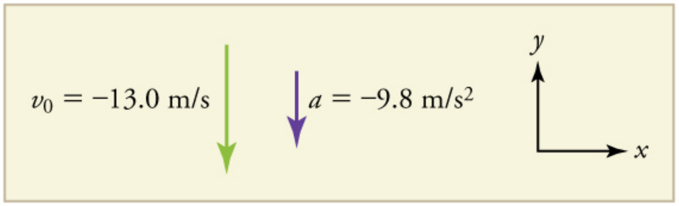
Example:

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.



Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

Equation:

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

Equation:

$$v = \pm 16.4\text{ m/s}.$$

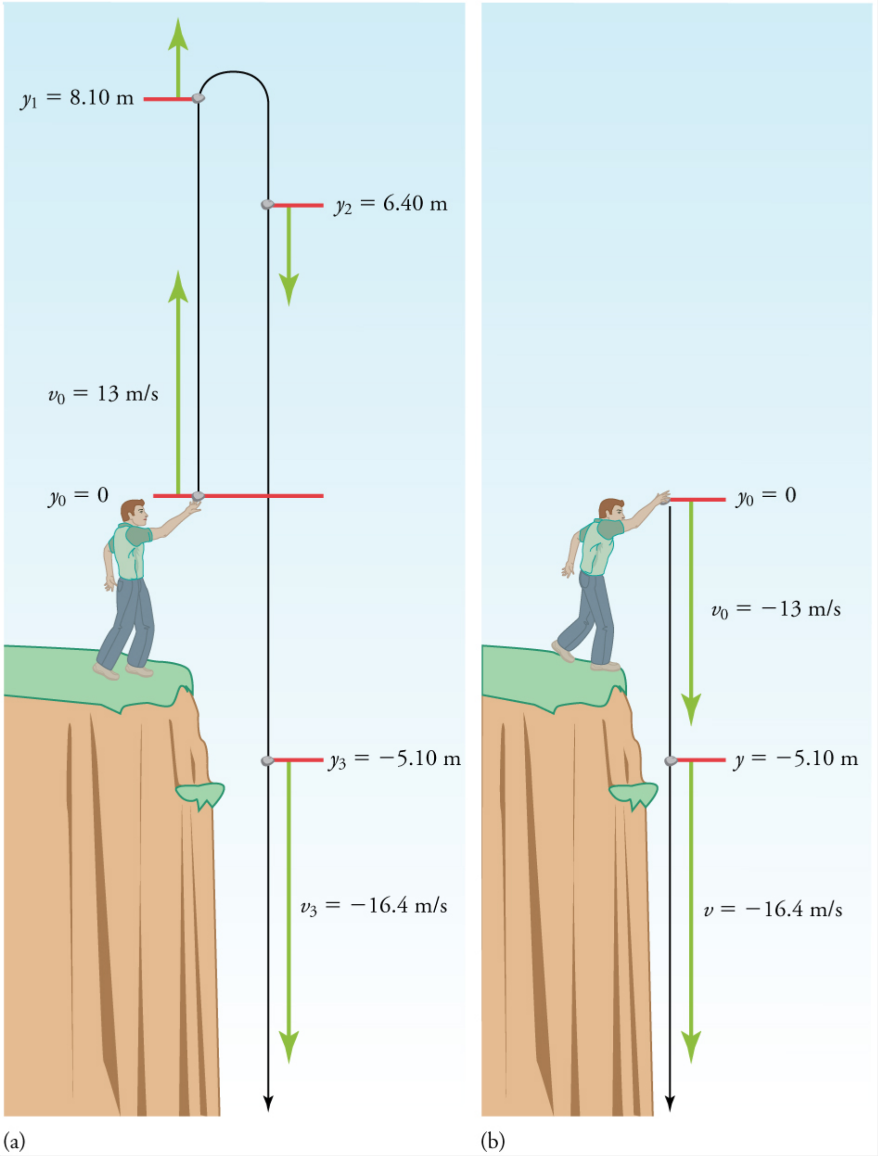
The negative root is chosen to indicate that the rock is still heading down. Thus,

Equation:

$$v = -16.4\ \text{m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See [link] and [link](a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from [link]) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20\ \text{m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.


$y_1 = 8.10$ m
$y_2 = 6.40$ m
$v_0 = 13$ m/s
$y_0 = 0$
$y_3 = -5.10$ m
$v_3 = -16.4$ m/s
(a)
$y_0 = 0$
$v_0 = -13$ m/s
$y = -5.10$ m
$v = -16.4$ m/s
(b)

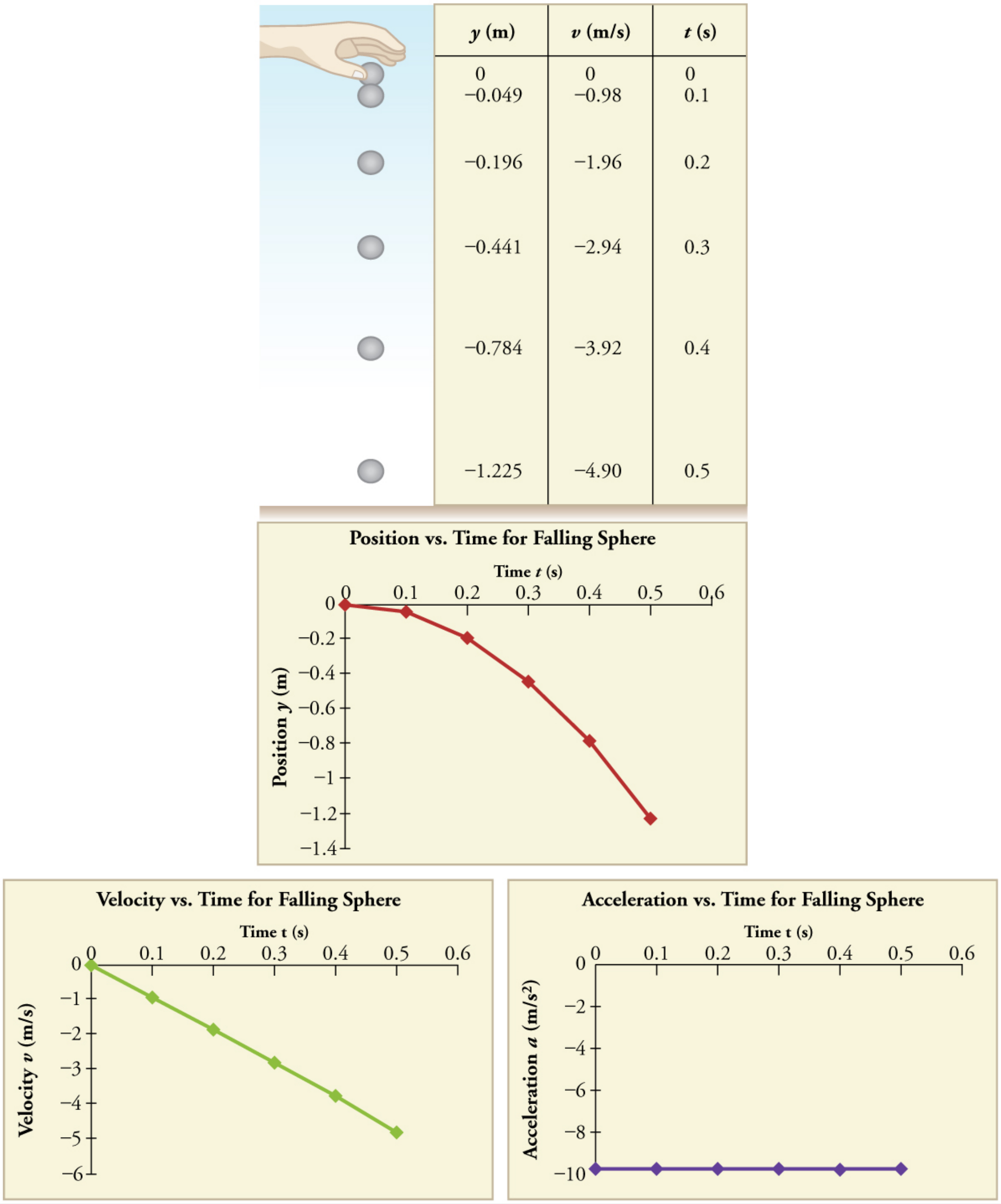
(a) A person throws a rock straight up, as explored in [link]. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in [link]. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In [link], the rock is thrown up with an initial velocity of $13.0\ \text{m}/\text{s}$. It rises and then falls back down. When its position is $y=0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y=-5.10 \text{ m}$ to be the same whether we have thrown it upwards at $+13.0 \text{ m/s}$ or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y=0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example:

Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, [link]. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



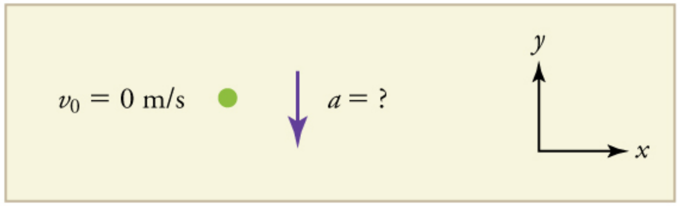
y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.



We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = –1.0000$ m; $t = 0.45173$; $v_0 = 0$.
2. Choose the equation that allows you to solve for a using the known values.

Equation:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

Equation:

$$y = y_0 + \frac{1}{2}at^2.$$

Solving for a gives

Equation:

$$a = \frac{2(y - y_0)}{t^2}.$$

4. Substitute known values yields

Equation:

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,

Equation:

$$g = 9.8010\ \text{m/s}^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80\ \text{m/s}^2$, so $9.8010\ \text{m/s}^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80\ \text{m/s}^2$; it represents the local value for the acceleration due to gravity.

Exercise:

Check Your Understanding

Problem:

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution:

We know that initial position $y_0 = 0$, final position $y = -30.0\ \text{m}$, and $a = -g = -9.80\ \text{m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

Equation:

$$\begin{aligned} y &= 0 + 0 - \tfrac{1}{2}gt^2 \\ t^2 &= \tfrac{2y}{-g} \\ t &= \pm\sqrt{\tfrac{2y}{-g}} = \pm\sqrt{\tfrac{2(-30.0\ \text{m})}{-9.80\ \text{m/s}^2}} = \pm\sqrt{6.12\ \text{s}^2} = 2.47\ \text{s} \approx 2.5\ \text{s} \end{aligned}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

Note:
PhET Explorations: Equation Grapher
Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = \text{bx}$) to see how they add to generate the polynomial curve.
https://phet.colorado.edu/sims/equation-grapher/equation-grapher_en.html

Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages
  Equation:
  $$g = 9.80\ \text{m/s}^2.$$
- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \text{m/s}^2$ is negative. In the opposite case, $a = +\text{g} = 9.80\ \text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Conceptual Questions

Exercise:

Problem:

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

Exercise:

Problem:

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

Exercise:

Problem:

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

Exercise:

Problem:

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

Exercise:

Problem:

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

Exercise:

Problem:

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

Problems & Exercises

Assume air resistance is negligible unless otherwise stated.

Exercise:

Problem:

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

Solution:

(a) $y_1 = 6.28 \text{ m}; v_1 = 10.1 \text{ m/s}$

(b) $y_2 = 10.1 \text{ m}; v_2 = 5.20 \text{ m/s}$

(c) $y_3 = 11.5 \text{ m}; v_3 = 0.300 \text{ m/s}$

(d) $y_4 = 10.4 \text{ m}; v_4 = -4.60 \text{ m/s}$

Exercise:

Problem:

Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

Exercise:

Problem:

A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

Solution:

$v_0 = 4.95\ \text{m/s}$

Exercise:

Problem:

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

Exercise:

Problem:

A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

Solution:

(a) $a = -9.80\ \text{m/s}^2$; $v_0 = 13.0\ \text{m/s}$; $y_0 = 0\ \text{m}$

(b) $v = 0\text{m/s}$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

Equation:

$$\begin{array}{lll} v^2 - v_0^2 & = & 2a(y - y_0) \\ \frac{v^2 - v_0^2}{2a} & = & y - y_0 \\ y & = & y_0 + \frac{v^2 - v_0^2}{2a} = 0\text{ m} + \frac{(0\text{ m/s})^2 - (13.0\text{ m/s})^2}{2\left(-9.80\text{ m/s}^2\right)} = 8.62\text{ m} \end{array}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c) 2.65 s

Exercise:

Problem:

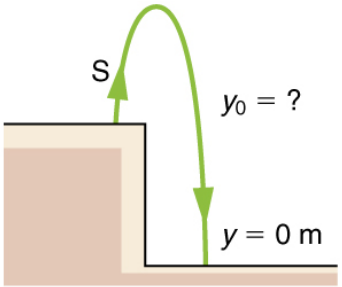
A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

Exercise:

Problem:

(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

Solution:



(a) 8.26 m

(b) 0.717 s

Exercise:

Problem:

A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

Exercise:

Problem:

You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

Solution:

1.91 s

Exercise:

Problem:

A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

Exercise:

Problem:

Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can’t see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

Solution:

(a) 94.0 m

(b) 3.13 s

Exercise:

Problem:

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

Exercise:

Problem:

There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

Solution:

(a) -70.0 m/s (downward)

(b) 6.10 s

Exercise:

Problem:

A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball’s initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

Exercise:

Problem:

Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

Solution:

(a) 19.6 m

(b) 18.5 m

Exercise:

Problem:

A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00\times 10^{-5}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Exercise:

Problem:

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

Solution:

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

Exercise:

Problem:

A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\text{ s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Glossary

free-fall	the state of movement that results from gravitational force only
acceleration due to gravity	acceleration of an object as a result of gravity

Graphical Analysis of One-Dimensional Motion

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

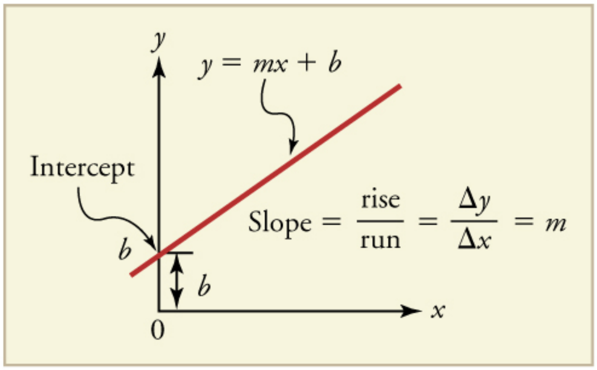
Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in [link], a straight-line graph has the general form

Equation:

$$y=\text{mx}+b.$$

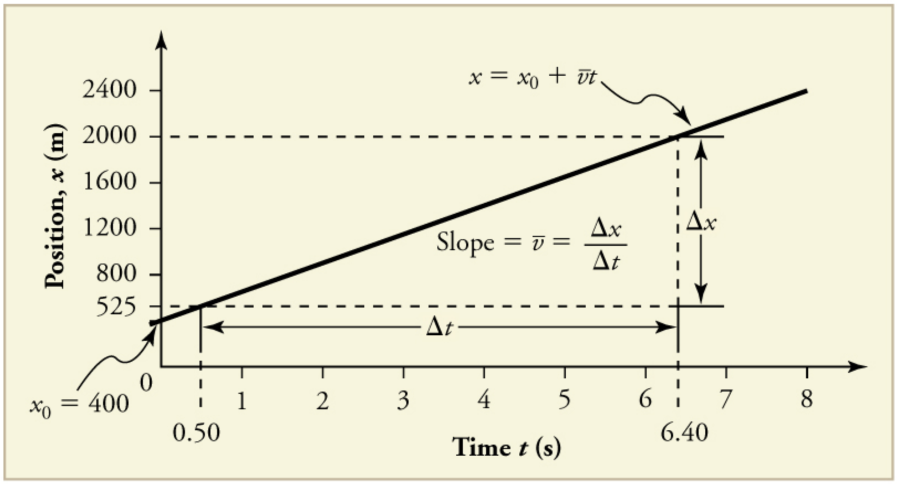
Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.



A straight-line graph. The equation for a straight line is $y=\text{mx}+b$.

Graph of Position vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. [link] is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.



Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is position at time zero—that is, x_0. Substituting these symbols into $y = \text{mx} + b$ gives

Equation:

$$x = \bar{v}t + x_0$$

or

Equation:

$$x = x_0 + \bar{v}t.$$

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

Note:

The Slope of *x* vs. *t*

The slope of the graph of position x vs. time t is velocity v.

Equation:

$$\text{slope} = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a position of 25 m at 0.50 s and 2000 m at 6.40 s. Its position at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example:

Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in [link].

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

Equation:

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}},$$

yielding

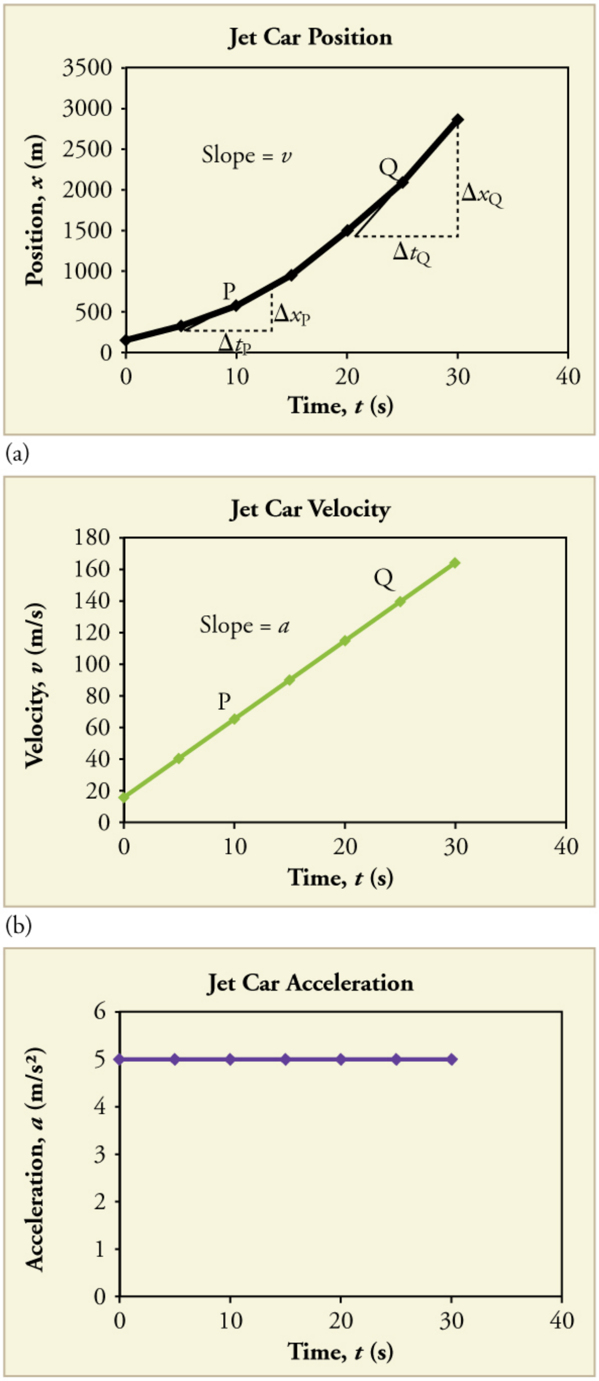
Equation:

$$\bar{v} = 250\text{ m/s}.$$

Discussion
This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in [link] below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.



(a)

(b)

(c)

Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of $5.0\ \text{m/s}^2$ over the time interval plotted.

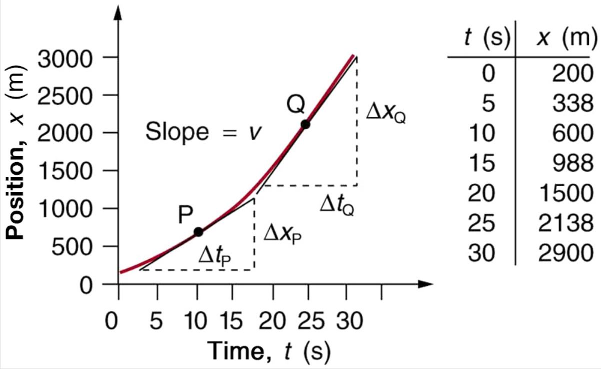
A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in [link](a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in [link](a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in [link](b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in [link](c).

Example:

Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in [link], where Q is the point at $t = 25$ s.

Solution

1. Find the tangent line to the curve at $t = 25$ s.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.
3. Plug these endpoints into the equation to solve for the slope, v.

Equation:

$$\text{slope} = v_{\text{Q}} = \frac{\Delta x_{\text{Q}}}{\Delta t_{\text{Q}}} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})}$$

Thus,

Equation:

$$v_{\text{Q}} = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}.$$

Discussion

This is the value given in this figure's table for v at $t = 25$ s. The value of 140 m/s for v_{Q} is plotted in [link]. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

Note:

The Slope of *v* vs. *t*

The slope of a graph of velocity v vs. time t is acceleration a.

Equation:

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in [link](b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in [link](c).

Additional general information can be obtained from [link] and the expression for a straight line, $y = \text{mx} + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

Equation:

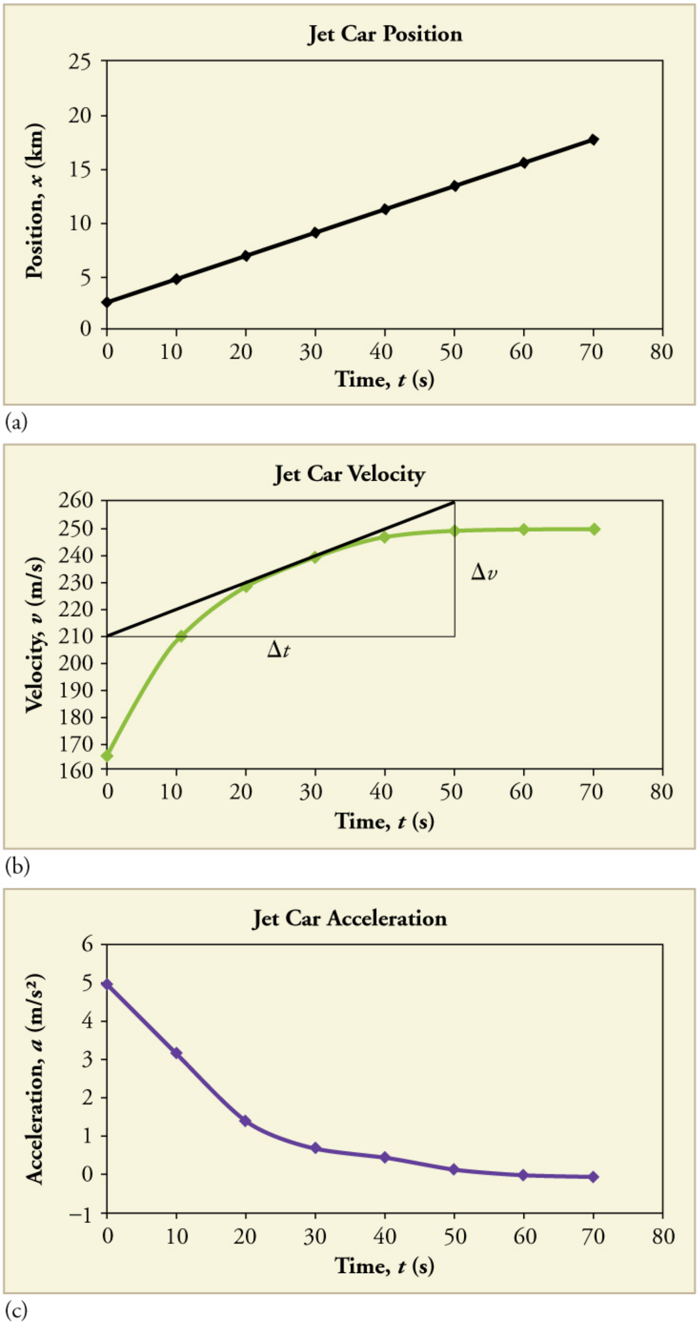
$$v = v_0 + \text{at}.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in [link]. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in [link].) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55$ s, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.



Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in

[link] ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example:

Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in [link](b).

Strategy

The slope of the curve at $t=25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in [link](b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

Equation:

$$\text{slope}=\frac{\Delta v}{\Delta t}=\frac{(260\text{ m/s}-210\text{ m/s})}{(51\text{ s}-1.0\text{ s})}$$

Equation:

$$a=\frac{50\text{ m/s}}{50\text{ s}}=1.0\text{ m/s}^2.$$

Discussion

Note that this value for a is consistent with the value plotted in [link](c) at $t=25\text{ s}$.

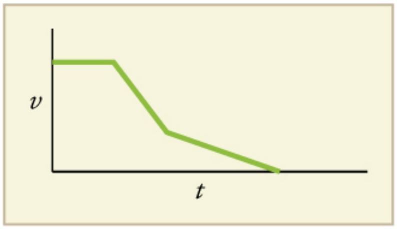
A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Exercise:

Check Your Understanding

Problem:

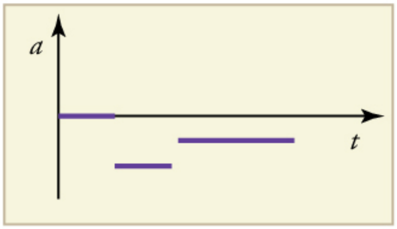
A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?



Solution:

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.



Section Summary

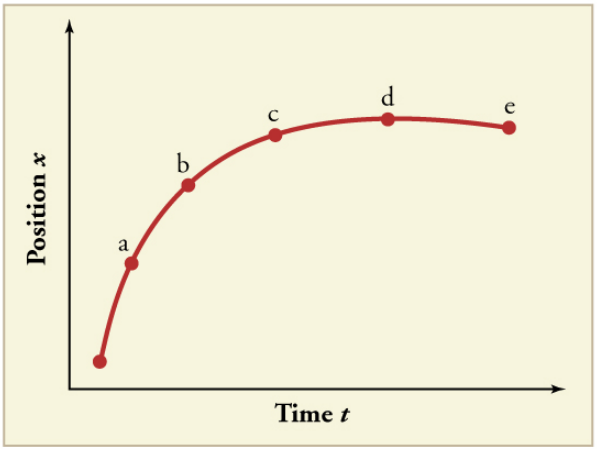
- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

Exercise:

Problem:

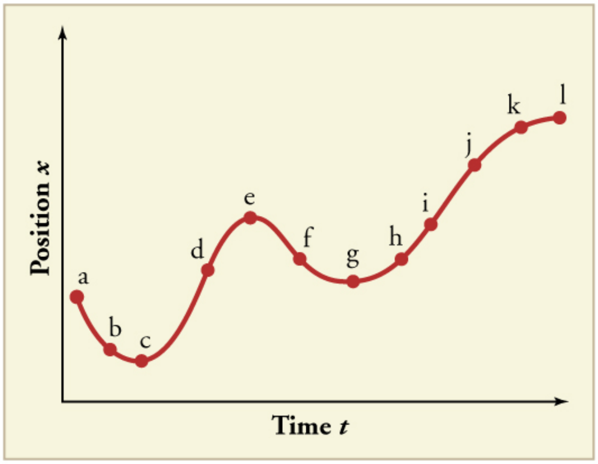
(a) Explain how you can use the graph of position versus time in [link] to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.



Exercise:

Problem:

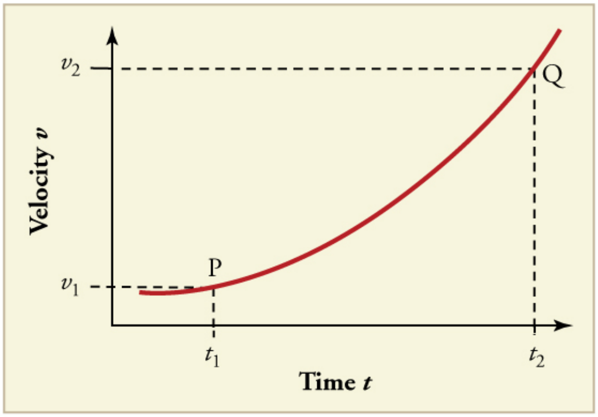
(a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in [link]. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?



Exercise:

Problem:

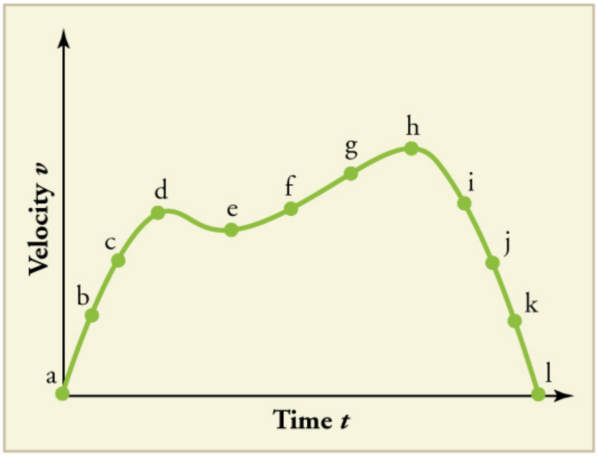
(a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in [link]. (b) Based on the graph, how does acceleration change over time?



Exercise:

Problem:

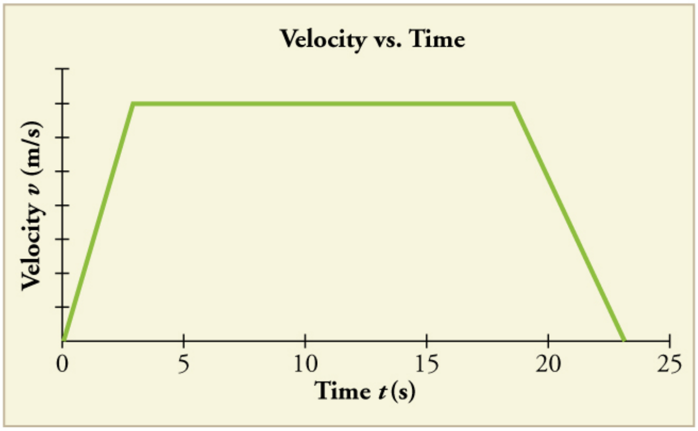
(a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in [link]. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?



Exercise:

Problem:

Consider the velocity vs. time graph of a person in an elevator shown in [link]. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.



Exercise:

Problem:

A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

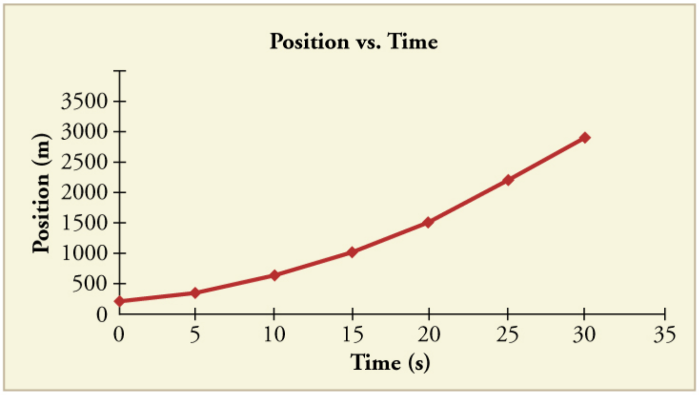
Problems & Exercises

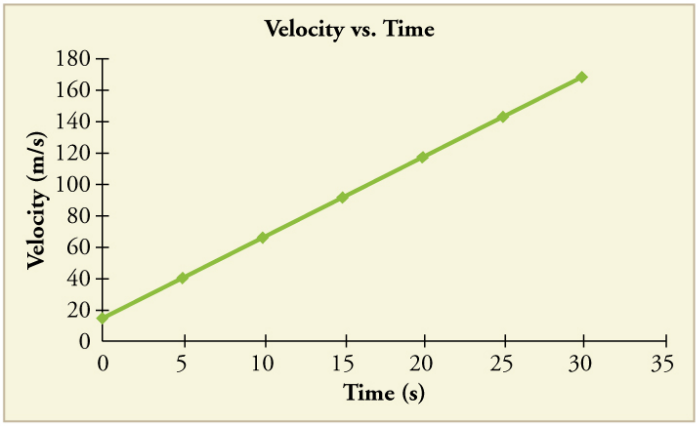
Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

Exercise:

Problem:

(a) By taking the slope of the curve in [link], verify that the velocity of the jet car is 115 m/s at $t = 20$ s. (b) By taking the slope of the curve at any point in [link], verify that the jet car's acceleration is $5.0\ \text{m/s}^2$.





Solution:

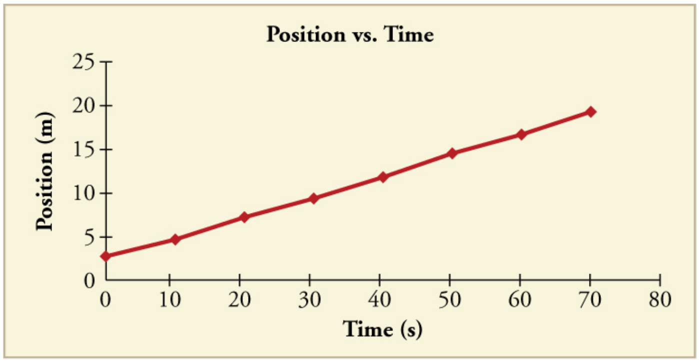
(a) $115\ \text{m/s}$

(b) $5.0\ \text{m/s}^2$

Exercise:

Problem:

Using approximate values, calculate the slope of the curve in [link] to verify that the velocity at $t = 10.0\ \text{s}$ is 0.208 m/s. Assume all values are known to 3 significant figures.



Exercise:

Problem:

Using approximate values, calculate the slope of the curve in [link] to verify that the velocity at $t = 30.0\ \text{s}$ is approximately 0.24 m/s.

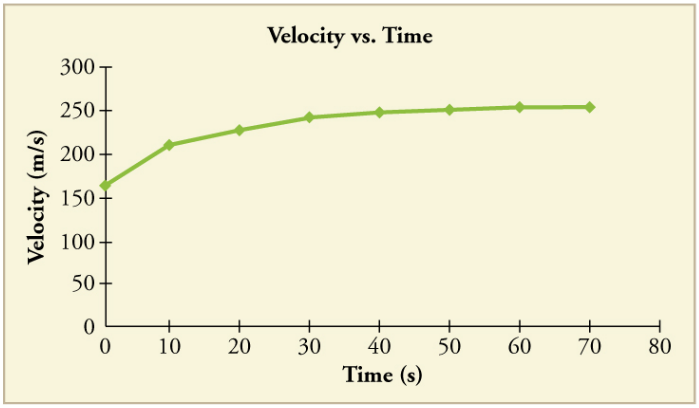
Solution:
Equation:

$$v=\frac{(11.7-6.95)\times 10^{3}\text{ m}}{(40.0-20.0)\text{ s}}=238\text{ m/s}$$

Exercise:

Problem:

By taking the slope of the curve in [link], verify that the acceleration is 3.2 m/s^{2} at $t=10$ s.

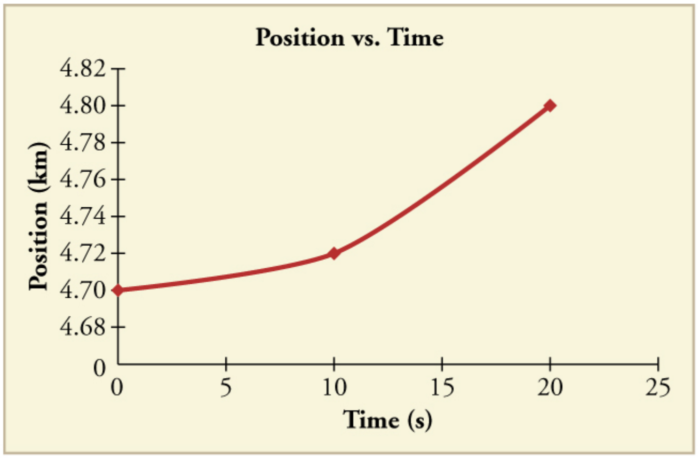


Exercise:

Problem:

Construct the position graph for the subway shuttle train as shown in [link](a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

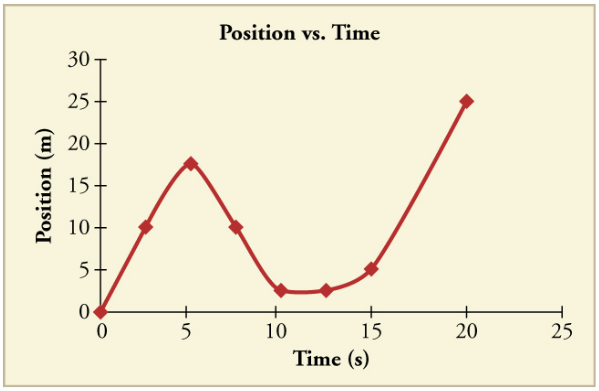
Solution:

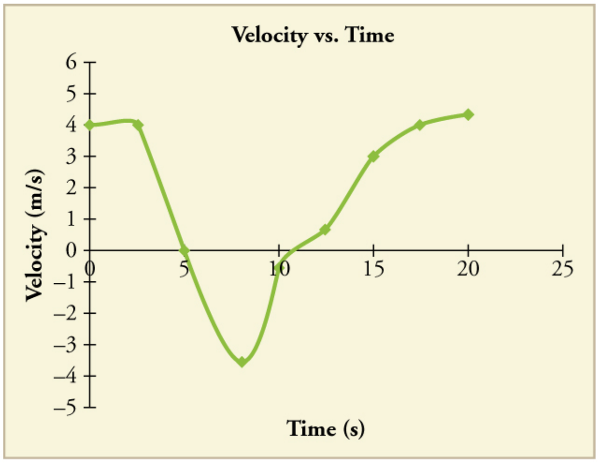


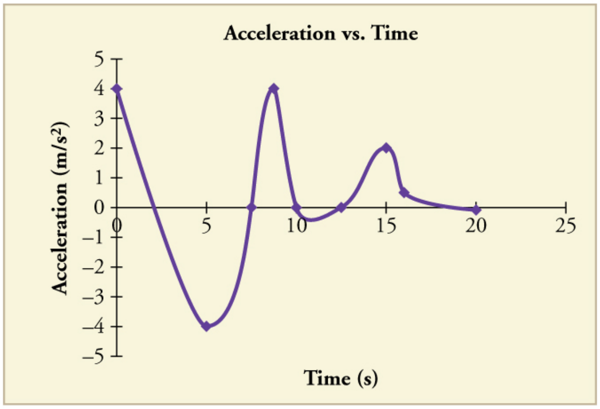
Exercise:

Problem:

(a) Take the slope of the curve in [link] to find the jogger’s velocity at $t = 2.5$ s. (b) Repeat at 7.5 s. These values must be consistent with the graph in [link].



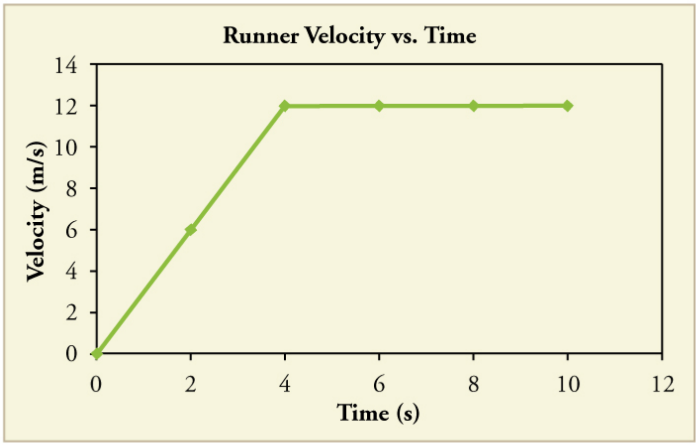




Exercise:

Problem:

A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See [link]). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5$ s? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?



Solution:

(a) 6 m/s

(b) 12 m/s

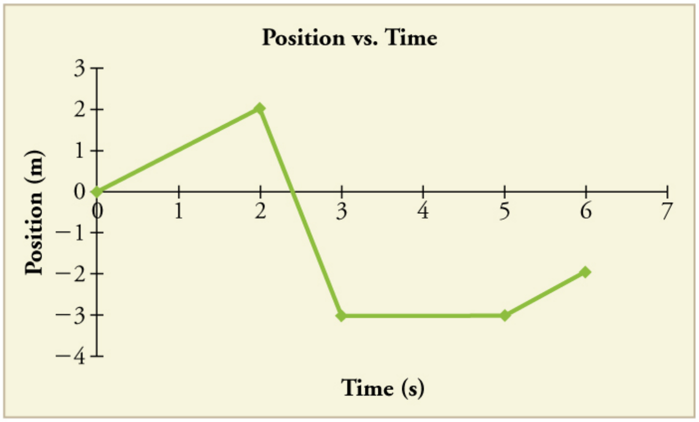
(c) $3\ \text{m/s}^2$

(d) 10 s

Exercise:

Problem:

[link] shows the position graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.



Glossary

independent variable
: the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable
: the variable that is being measured; usually plotted along the y-axis

slope
: the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept
: the y-value when x= 0, or when the graph crosses the y-axis

Introduction to Two-Dimensional Kinematics class="introduction"

Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain’s Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

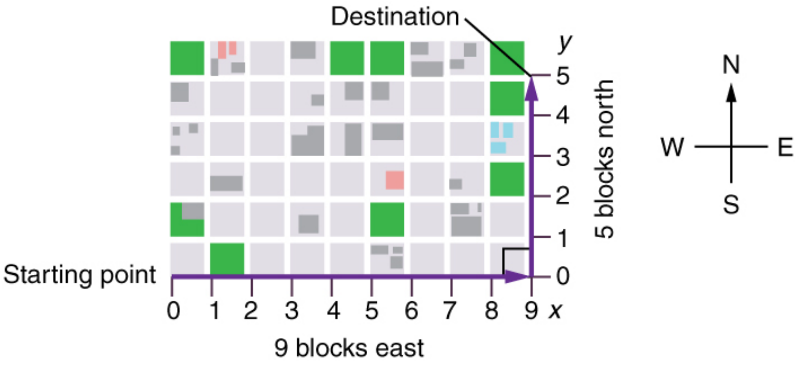
Kinematics in Two Dimensions: An Introduction

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in [link].

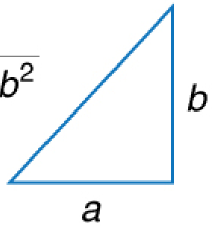

Destination
y
5
4
3
2
1
0
5 blocks north
N
W
E
S
Starting point
0
1
2
3
4
5
6
7
8
9
x
9 blocks east

A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

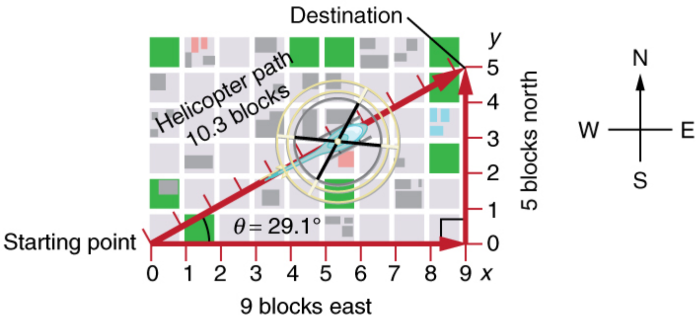
An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

$$c = \sqrt{a^2 + b^2}$$



The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9 \text{ blocks})^2 + (5 \text{ blocks})^2} = 10.3 \text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)


Destination
y
5
4
3
2
1
0
5 blocks north
Helicopter path
10.3 blocks
$\theta = 29.1°$
Starting point
0
1
2
3
4
5
6
7
8
9
x
9 blocks east
N
W
E
S

The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in [link] is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

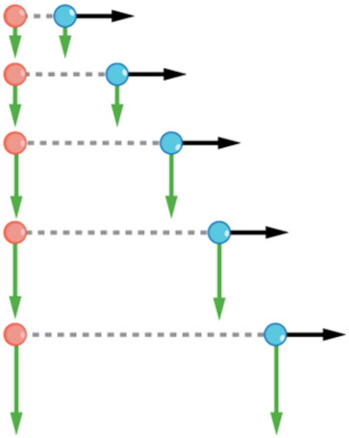
As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in [link] and [link]. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in [link]. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in [link] walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Note:
Independence of Motion
The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let’s compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.



This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

Note:
PhET Explorations: Ladybug Motion 2D
Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.
https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/#sim-ladybug-motion

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion

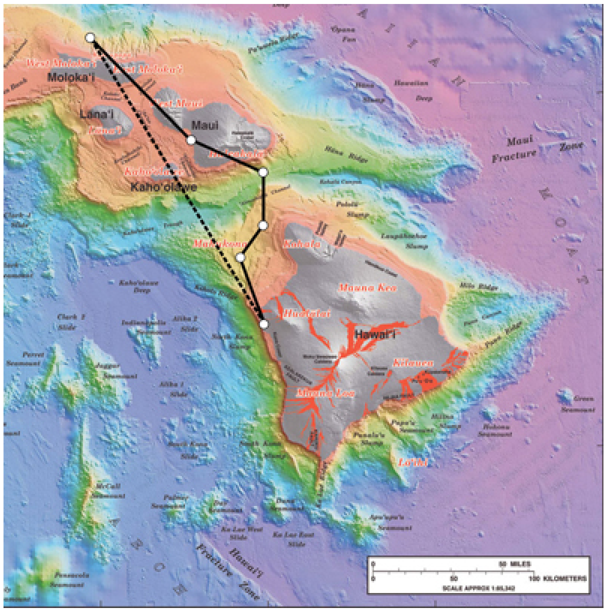
in the vertical direction, and vice versa.

Glossary

vector	a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

Vector Addition and Subtraction: Graphical Methods

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.


West Moloka'i
East Moloka'i
Moloka'i
Lana'i
Lana'i
West Maui
Maui
Haleakala
Kaho'olawe
Kaho'olawe
Mahukona
Kohala
Mauna Kea
Hualalai
Hawai'i
Kilauea
Mauna Loa
Lo'ihi
Maui Fracture Zone
Hawai'i Fracture Zone
Hawaiian Deep
Kaho'olawe Deep
Hilo Ridge
Puna Ridge
Hana Ridge
Clark 1 Slide
Alika 1 Slide
Alika 2 Slide
North Kona Slump
South Kona Slide
South Kona Slump
Ka Lae West Slide
Ka Lae East Slide
Pololu Slump
Laupahoehoe Slump
Hilina Slump
Punalu'u Slump
Papa'u Seamount
Apo'apo'a Seamount
Green Seamount
Hohonu Seamount
Jaggar Seamount
Day Seamount
Dana Seamount
Palmer Seamount
McCall Seamount
Pensacola Seamount
Indianapolis Seamount
Perret Seamount
'Opana Fan
Pu'uwai Ridge
0 50 MILES
0 50 100 KILOMETERS
SCALE APPROX 1:85,342

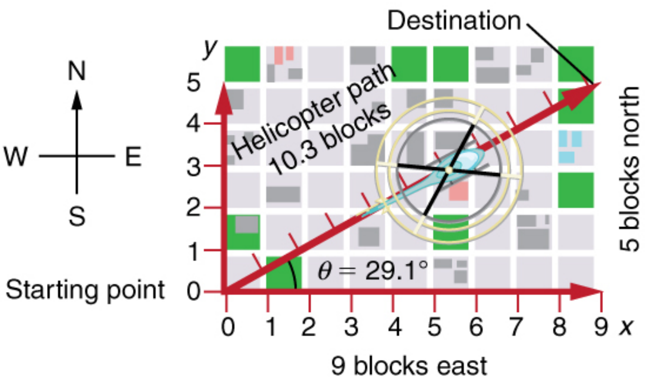
Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

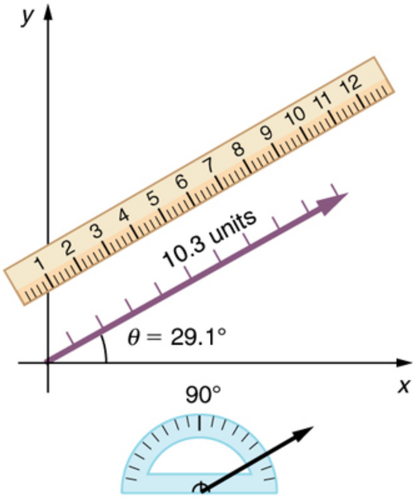
A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

[link] shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Note:
Vectors in this Text
In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.



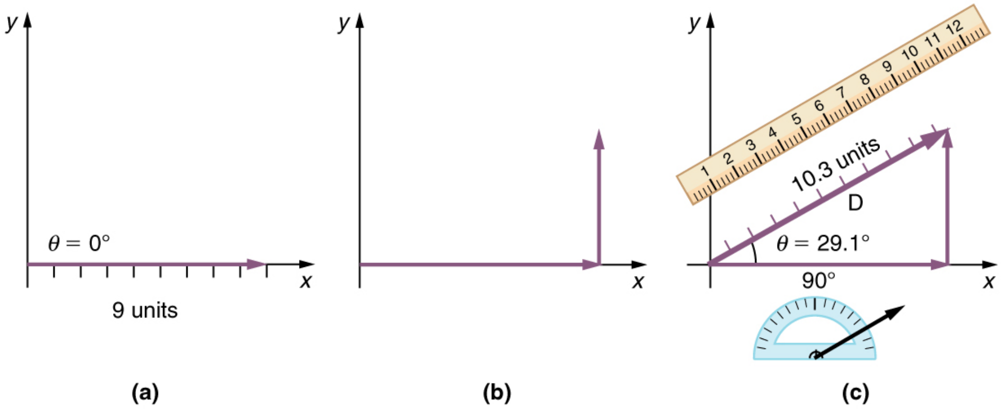
A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.



To describe the resultant vector for the person walking in a city considered in [link] graphically, draw an arrow to represent the total displacement vector D. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

Vector Addition: Head-to-Tail Method

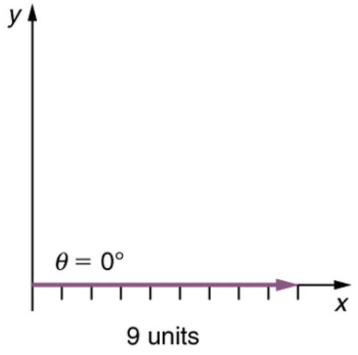
The **head-to-tail method** is a graphical way to add vectors, described in [link] below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.



Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in [link]. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector.

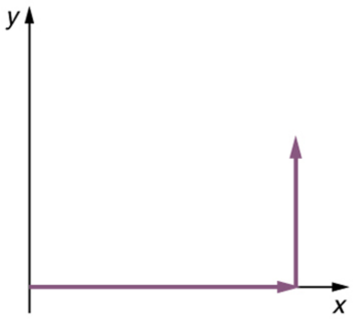
(c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1º.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.



(a)

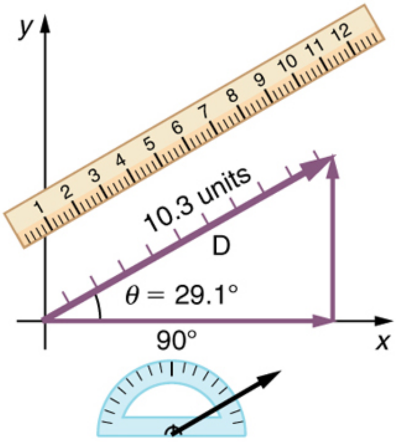
Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.



(b)

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.



(c)

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example:

Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

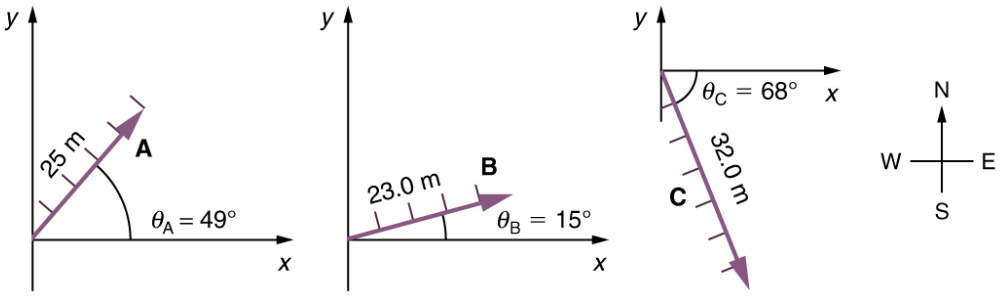
Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0º north of east. Then, she walks 23.0 m heading 15.0º north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first A, the second B, and the third C, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted **R**.

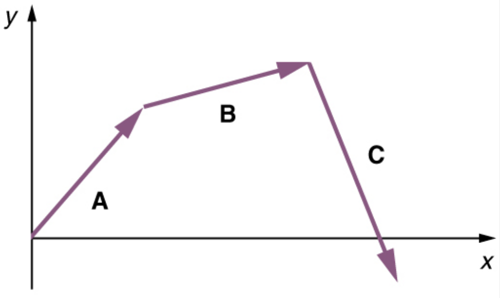
Solution

(1) Draw the three displacement vectors.



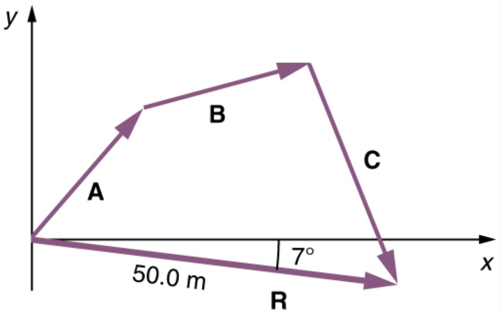
(a)

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

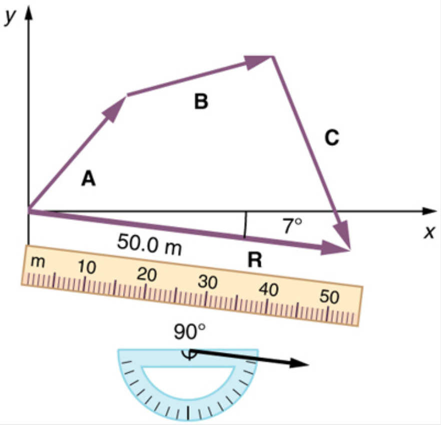


(a)

(3) Draw the resultant vector, R.



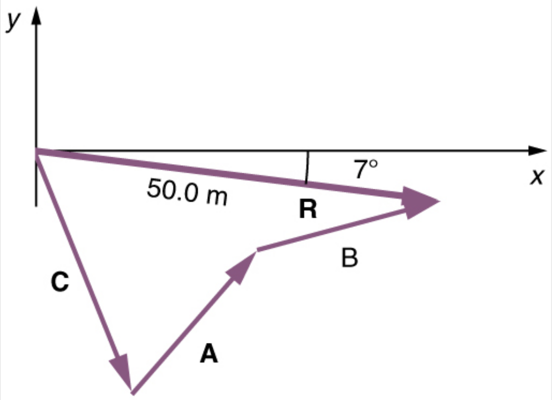
(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of R. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.



In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0\text{ m}$ and $\theta = 7.0^\circ$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in [link] and we will still get the same solution.



Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

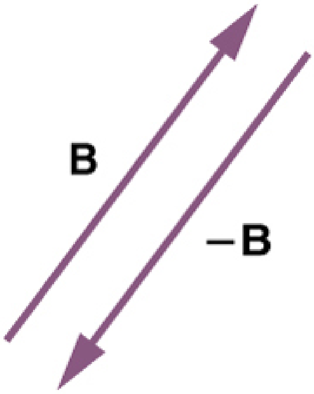
Equation:

$$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2}+\mathbf{3}$ or $\mathbf{3}+\mathbf{2}$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A}–\mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $–\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in [link]. In other words, $\mathbf{B}$ has the same length as $–\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.



The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $-\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

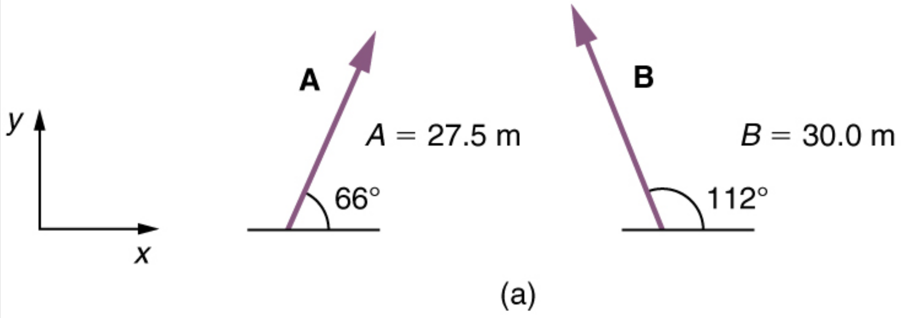
Equation:

$$\mathrm{A} - \mathrm{B} = \mathrm{A} + (-\mathbf{B}).$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

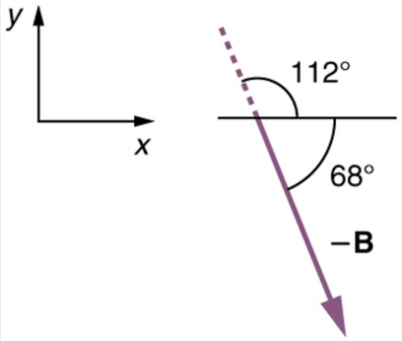
Example:
Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.



Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A}+\mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^{\circ}-112^{\circ}=68^{\circ}$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A}+(-\mathbf{B})$, or $\mathbf{A}-\mathbf{B}$.



We will perform vector addition to compare the location of the dock, $\mathrm{A} + \mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathrm{A} + (–\mathrm{B})$.

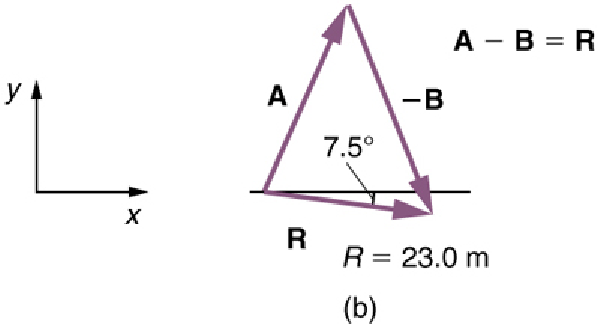
Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

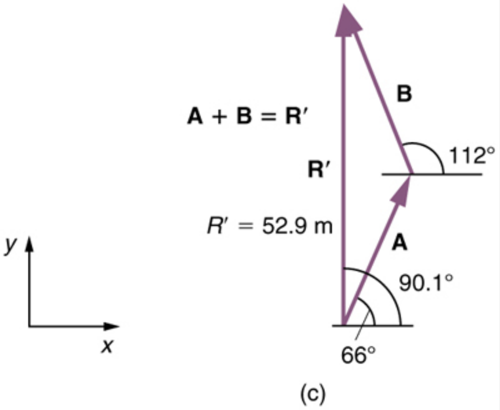
(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.



In this case, $R = 23.0\text{ m}$ and $\theta = 7.5^{\text{o}}$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}\prime$:



In this case $R = 52.9\text{ m}$ and $\theta = 90.1^{\text{o}}$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5 \text{ m}$, or 82.5 m, in a direction 66.0° north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5 \text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y-*components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0° north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton’s Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

Note:
PhET Explorations: Maze Game
Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.
https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game

Summary

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at

the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by $c\mathrm{A}$. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

Conceptual Questions

Exercise:

Problem:

Which of the following is a vector: a person’s height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth’s population, the acceleration of gravity?

Exercise:

Problem:

Give a specific example of a vector, stating its magnitude, units, and direction.

Exercise:

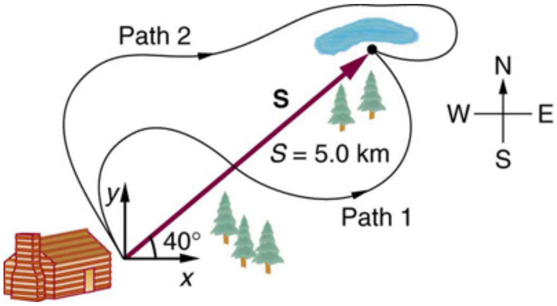
Problem:

What do vectors and scalars have in common? How do they differ?

Exercise:

Problem:

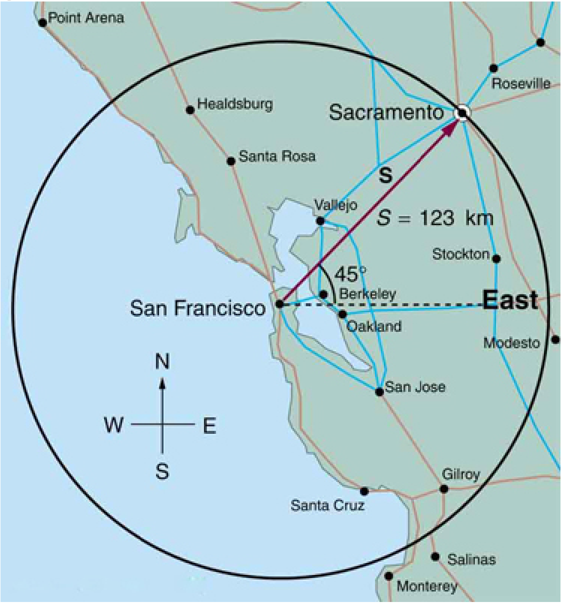
Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?


Path 2
S
S = 5.0 km
Path 1
y
40°
x
N
W
E
S

Exercise:

Problem:

If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in [link]. What other information would he need to get to Sacramento?


Point Arena
Healdsburg
Roseville
Sacramento
Santa Rosa
S
Vallejo
S = 123 km
Stockton
45°
Berkeley
San Francisco
East
Oakland
Modesto
N
San Jose
W
E
S
Gilroy
Santa Cruz
Salinas
Monterey

Exercise:

Problem:

Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A}+\mathbf{B}$ the sum of the lengths of the two steps?

Exercise:

Problem: Explain why it is not possible to add a scalar to a vector.

Exercise:

Problem:

If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

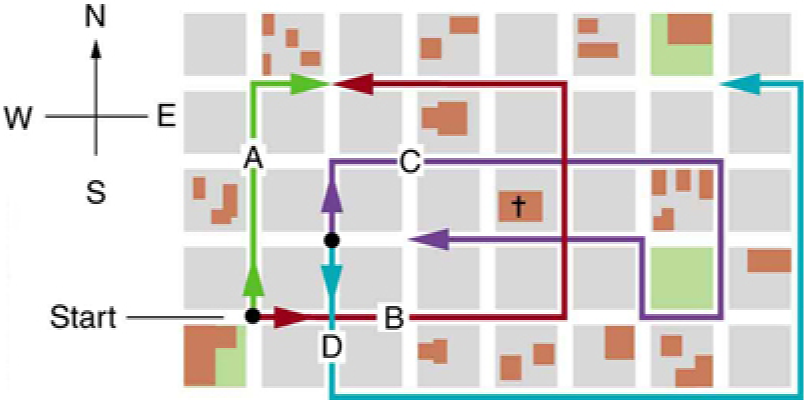
Problems & Exercises

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

Exercise:

Problem:

Find the following for path A in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution:

(a) 480 m

(b) 379 m, 18.4º east of north

Exercise:

Problem:

Find the following for path B in [link]: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

Exercise:

Problem:

Find the north and east components of the displacement for the hikers shown in [link].

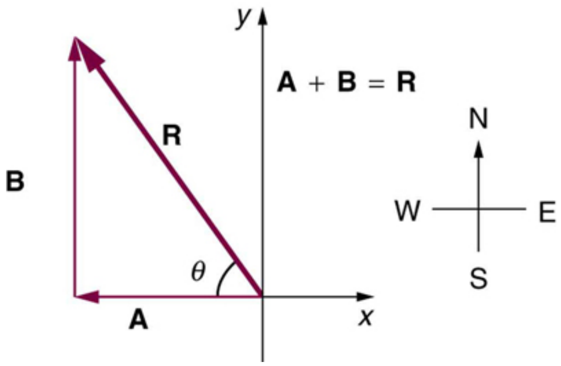
Solution:

north component 3.21 km, east component 3.83 km

Exercise:

Problem:

Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

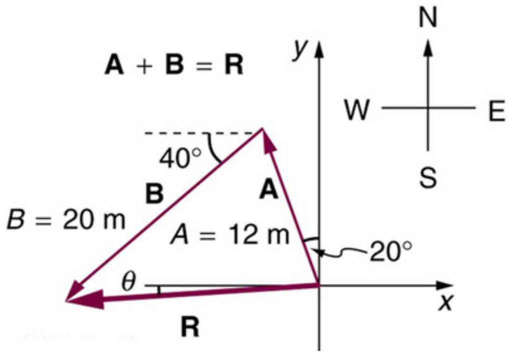


The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Exercise:

Problem:

Suppose you first walk 12.0 m in a direction 20º west of north and then 20.0 m in a direction 40.0º south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem finds their sum $\mathrm{R} = \mathrm{A} + \mathrm{B}$.)



Solution:

19.5 m, 4.65º south of west

Exercise:

Problem:

Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40º south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20º west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

Exercise:

Problem:

(a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.

Solution:

(a) $26.6\text{ m}, 65.1^\circ$ north of east

(b) $26.6\text{ m}, 65.1^\circ$ south of west

Exercise:

Problem:

Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathrm{A} + \mathrm{B} + \mathrm{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

Exercise:

Problem:

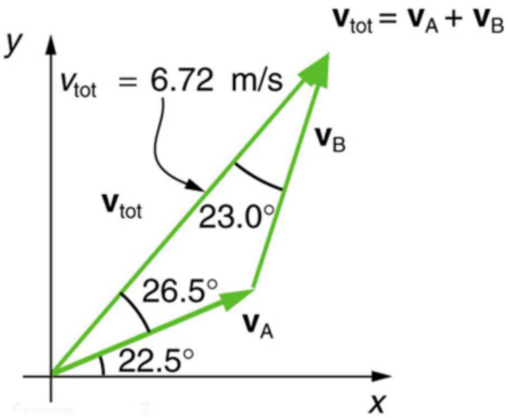
Show that the sum of the vectors discussed in [link] gives the result shown in [link].

Solution:

52.9 m, 90.1° with respect to the *x*-axis.

Exercise:

Problem: Find the magnitudes of velocities v_A and v_B in [link]


$\mathbf{v}_{tot} = \mathbf{v}_A + \mathbf{v}_B$
y
v_{tot} = 6.72 m/s
$\mathbf{v}_B$
$\mathbf{v}_{tot}$
23.0°
26.5°
$\mathbf{v}_A$
22.5°
x

The two velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ add to give a total $\mathbf{v}_{tot}$.

Exercise:

Problem:

Find the components of v_{tot} along the *x*- and *y*-axes in [link].

Solution:

x-component 4.41 m/s

y-component 5.07 m/s

Exercise:

Problem:

Find the components of v_{tot} along a set of perpendicular axes rotated 30^{o} counterclockwise relative to those in [link].

Glossary

component (of a 2-d vector)
: a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative
: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector)
: the orientation of a vector in space

head (of a vector)
: the end point of a vector; the location of the tip of the vector’s arrowhead; also referred to as the “tip”

head-to-tail method
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector)
: the length or size of a vector; magnitude is a scalar quantity

resultant
: the sum of two or more vectors

resultant vector	the vector sum of two or more vectors
scalar	a quantity with magnitude but no direction
tail	the start point of a vector; opposite to the head or tip of the arrow

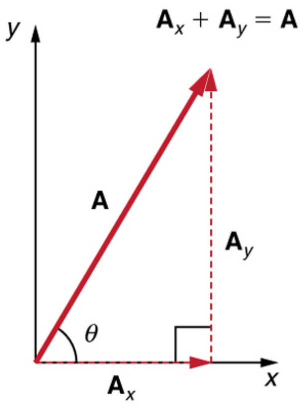
Vector Addition and Subtraction: Analytical Methods

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in [link], we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.



The vector $\mathbf{A}$, with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the *x*- and *y*-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

Equation:

$$\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\ \mathrm{m}$ east, $\mathbf{A}_y = 4\ \mathrm{m}$ north, and $\mathbf{A} = 5\ \mathrm{m}$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathrm{A}_y = \mathrm{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

Equation:

$$3\text{ m} + 4\text{ m} \ \neq \ 5\text{ m}$$

Thus,

Equation:

$$A_x + A_y \neq A$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*- and *y*-components, we use the following relationships for a right triangle.

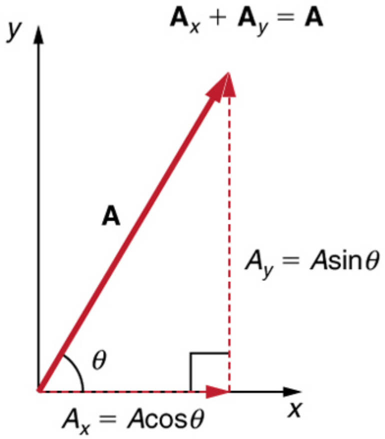
Equation:

$$A_x = A \cos \theta$$

and

Equation:

$$A_y = A \sin \theta.$$

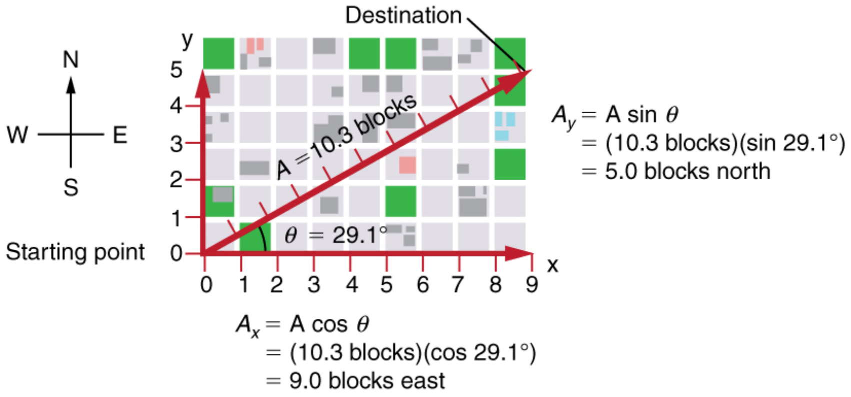


$\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$
y
$\mathbf{A}$
$A_y = A\sin\theta$
θ
$A_x = A\cos\theta$
x

The magnitudes of the vector

components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin \theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.


Destination
N
W
E
S
Starting point
y
x
0
1
2
3
4
5
0
1
2
3
4
5
6
7
8
9
A = 10.3 blocks
θ = 29.1°
A_y = A sin θ
= (10.3 blocks)(sin 29.1°)
= 5.0 blocks north
A_x = A cos θ
= (10.3 blocks)(cos 29.1°)
= 9.0 blocks east

We can use the relationships $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^{\text{o}}$, so that

Equation:

$$A_x = A\ \text{cos}\ \theta = (10.3\ \text{blocks})(\text{cos}\ 29.1^{\text{o}}) = 9.0\ \text{blocks}$$

Equation:

$$A_y = A\ \text{sin}\ \theta = (10.3\ \text{blocks})(\text{sin}\ 29.1^{\text{o}}) = 5.0\ \text{blocks}.$$

Calculating a Resultant Vector

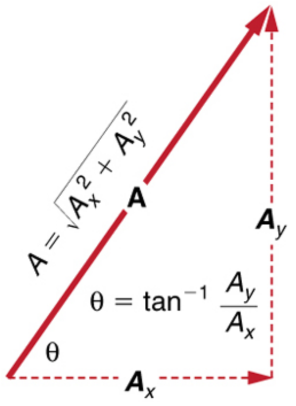
If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, we use the following relationships:

Equation:

$$A = \sqrt{A_{x^2} + A_{y^2}}$$

Equation:

$$\theta = \text{tan}^{-1}(A_y/A_x).$$


$A = \sqrt{A_x^2 + A_y^2}$
A
A_y
$\theta = \tan^{-1} \frac{A_y}{A_x}$
θ
A_x

The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2+5^2}=10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9)=29.1^{\circ}$, as before.

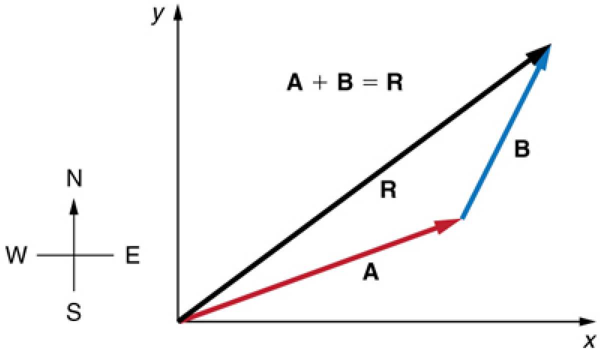
Note:

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

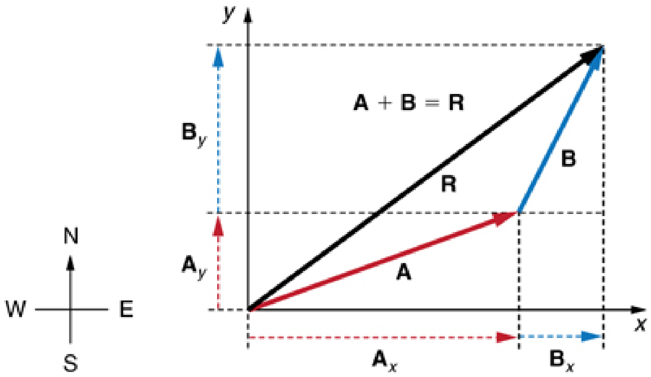
To see how to add vectors using perpendicular components, consider [link], in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.



Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{{A_x}^2 + {A_y}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen* *perpendicular axes*. Use the equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to find the components. In [link], these components are A_x, A_y, B_x, and B_y. The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B, respectively.



To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis*. That is, as shown in [link],

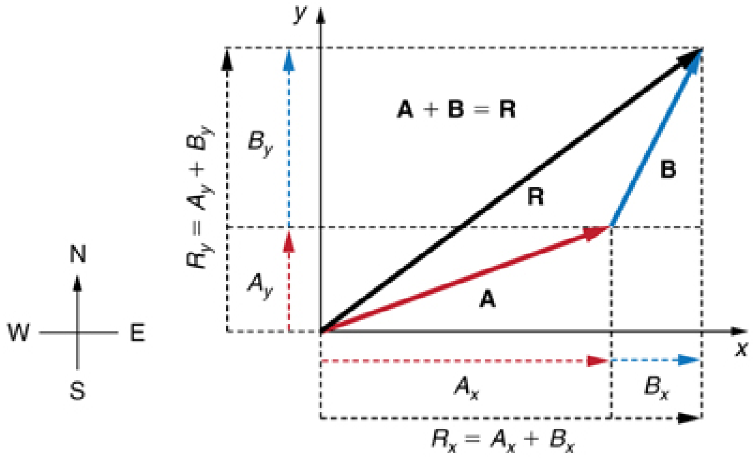
Equation:

$$R_x = A_x + B_x$$

and

Equation:

$$R_y = A_y + B_y.$$



The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

Equation:

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4. *To get the direction of the resultant:*
Equation:

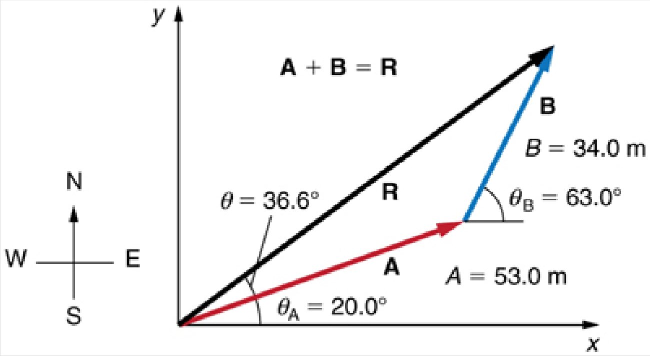
$$\theta = \tan^{-1}(R_y/R_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example:

Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in [link], using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0° north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of 34.0 m in a direction 63.0° north of east.



Vector $\mathbf{A}$ has magnitude 53.0 m and direction 20.0° north of the *x*-axis. Vector $\mathbf{B}$ has magnitude 34.0 m and direction 63.0° north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. Note that $A = 53.0\text{ m}$, $\theta_\text{A} = 20.0^\text{o}$, $B = 34.0\text{ m}$, and $\theta_\text{B} = 63.0^\text{o}$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

Equation:

$$\begin{aligned} A_x &= A\cos\theta_\text{A} = (53.0\text{ m})(\cos 20.0^\text{o}) \\ &= (53.0\text{ m})(0.940) = 49.8\text{ m} \end{aligned}$$

and

Equation:

$$\begin{aligned} B_x &= B\cos\theta_\text{B} = (34.0\text{ m})(\cos 63.0^\text{o}) \\ &= (34.0\text{ m})(0.454) = 15.4\text{ m}. \end{aligned}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_\text{A}$:

Equation:

$$\begin{aligned} A_y &= A\sin\theta_\text{A} = (53.0\text{ m})(\sin 20.0^\text{o}) \\ &= (53.0\text{ m})(0.342) = 18.1\text{ m} \end{aligned}$$

and

Equation:

$$\begin{aligned} B_y &= B\sin\theta_\text{B} = (34.0\text{ m})(\sin 63.0^\text{o}) \\ &= (34.0\text{ m})(0.891) = 30.3\text{ m}. \end{aligned}$$

The *x*- and *y*-components of the resultant are thus

Equation:

$$R_x = A_x + B_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m}$$

and

Equation:

$$R_y = A_y + B_y = 18.1\text{ m} + 30.3\text{ m} = 48.4\text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

Equation:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2\text{ m}}$$

so that

Equation:

$$R = 81.2\text{ m}.$$

Finally, we find the direction of the resultant:

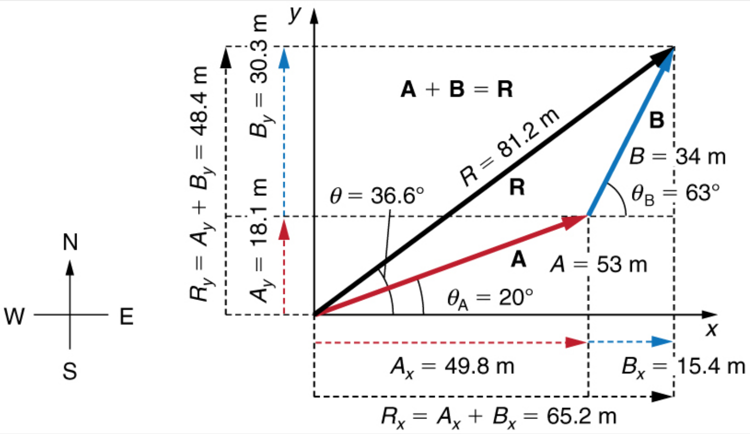
Equation:

$$\theta = \tan^{-1}(R_y/R_x) {=} {+}\tan^{-1}(48.4/65.2).$$

Thus,

Equation:

$$\theta = \tan^{-1}(0.742) = 36.6^{\circ}.$$



y
x
N
W
E
S
A + B = R
R = 81.2 m
R
B
B = 34 m
$\theta_B = 63^{\circ}$
$\theta = 36.6^{\circ}$
A
A = 53 m
$\theta_A = 20^{\circ}$
$A_y = 18.1$ m
$B_y = 30.3$ m
$R_y = A_y + B_y = 48.4$ m
$A_x = 49.8$ m
$B_x = 15.4$ m
$R_x = A_x + B_x = 65.2$ m

Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*- and *y*-components of the resultant $\mathbf{A} - \mathrm{B} = \mathrm{R}$ are thus

Equation:

$$R_x = A_x + (-B_x)$$

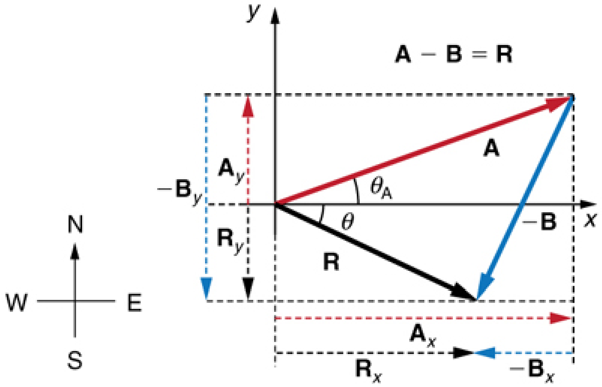
and

Equation:

$$R_y = A_y + (-B_y)$$

and the rest of the method outlined above is identical to that for addition. (See [link].)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.



The subtraction of the two vectors shown in [link]. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

Note:
PhET Explorations: Vector Addition
Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.
https://phet.colorado.edu/sims/vector-addition/vector-addition_en.html

Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

Equation:

$$\begin{aligned} A_x &= A \cos \theta \\ B_x &= B \cos \theta \end{aligned}$$

and

Equation:

$$\begin{aligned} A_y &= A \sin \theta \\ B_y &= B \sin \theta. \end{aligned}$$

Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

Equation:

$$R_x = A_x + B_x$$

and

Equation:

$$R_y = A_y + B_{y.}$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

Equation:

$$R = \sqrt{R_x^2 + R_y^2}.$$

Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$

:

Equation:

$$\theta = \tan^{-1}(R_y/R_x).$$

Conceptual Questions

Exercise:

Problem:

Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

Exercise:

Problem:

Give an example of a nonzero vector that has a component of zero.

Exercise:

Problem:

Explain why a vector cannot have a component greater than its own magnitude.

Exercise:

Problem:

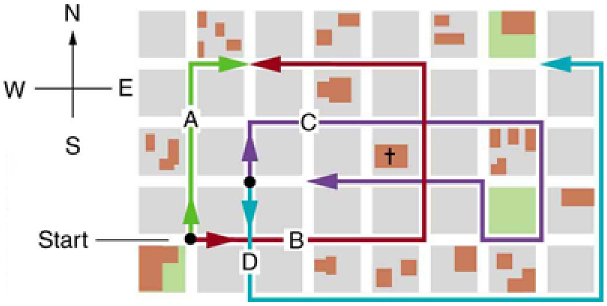
If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

Problems & Exercises

Exercise:

Problem:

Find the following for path C in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution:

(a) 1.56 km

(b) 120 m east

Exercise:

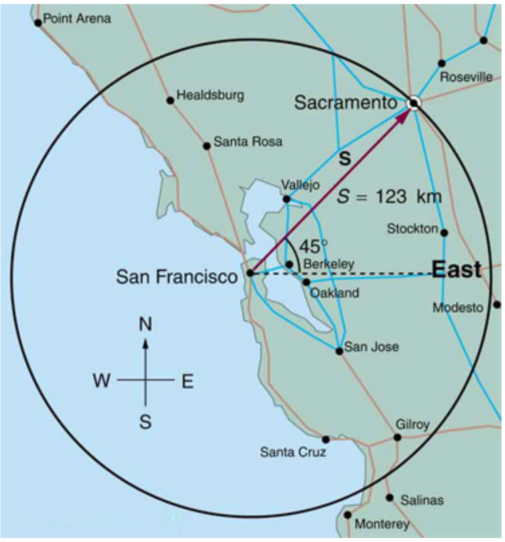
Problem:

Find the following for path D in [link]: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

Exercise:

Problem:

Find the north and east components of the displacement from San Francisco to Sacramento shown in [link].


Point Arena
Roseville
Healdsburg
Sacramento
Santa Rosa
S
Vallejo
S = 123 km
Stockton
45°
Berkeley
San Francisco
East
Oakland
Modesto
N
W
E
S
San Jose
Gilroy
Santa Cruz
Salinas
Monterey

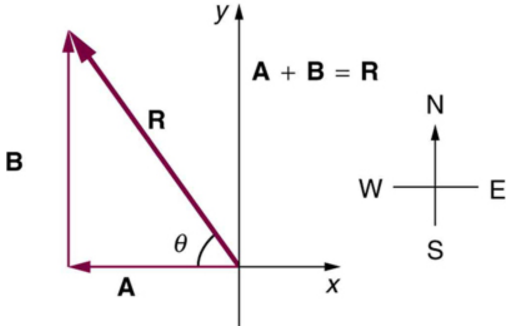
Solution:

North-component 87.0 km, east-component 87.0 km

Exercise:

Problem:

Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in [link], then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)



The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

Exercise:

Problem:

Repeat [link] using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result —that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

Solution:

30.8 m, 35.8 west of north

Exercise:

Problem:

You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

Exercise:

Problem:

Do [link] again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R'} = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

Solution:

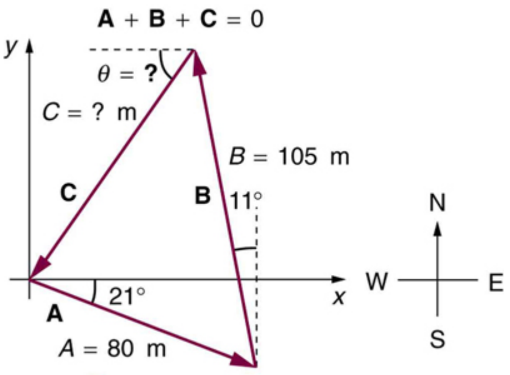
(a) 30.8 m, 54.2° south of west

(b) 30.8 m, 54.2° north of east

Exercise:

Problem:

A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in [link]. She then correctly calculates the length and orientation of the third side C. What is her result?



Exercise:

Problem:

You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

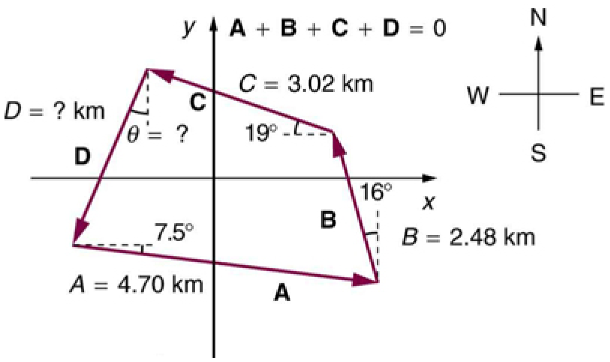
Solution:

18.4 km south, then 26.2 km west(b) 31.5 km at 45.0° south of west, then 5.56 km at 45.0° west of north

Exercise:

Problem:

A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in [link], and then correctly calculates the length and orientation of the fourth side $\mathbf{D}$. What is his result?



Exercise:

Problem:

In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: 2.50 km 45.0° north of west; then 4.70 km 60.0° south of east; then 1.30 km 25.0° south of west; then 5.10 km straight east; then 1.70 km 5.00° east of north; then 7.20 km 55.0° south of west; and finally 2.80 km 10.0° north of east. What is his final position relative to the island?

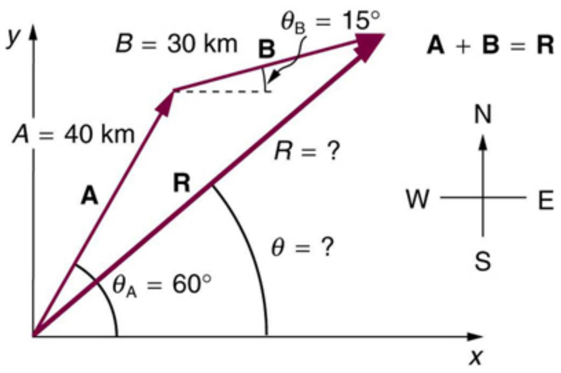
Solution:

7.34 km, 63.5° south of east

Exercise:

Problem:

Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in [link]. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



$\theta_B = 15°$
y
$B = 30$ km
B
A + **B** = **R**
N
$A = 40$ km
$R = ?$
A
R
W
E
$\theta = ?$
S
$\theta_A = 60°$
x

Glossary

analytical method

the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

Projectile Motion

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. [link] illustrates the notation for displacement, where **s** is defined to be the total displacement and **x** and **y** are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are *s*, *x*, and *y*. (Note that in the last section we used the notation $\mathbf{A}$ to represent a vector with components $\mathbf{A}_x$ and $\mathbf{A}_y$. If we continued this format, we would call displacement **s** with components $\mathbf{s}_x$ and $\mathbf{s}_y$. However, to simplify the notation, we will simply represent the component vectors as $\mathbf{x}$ and $\mathbf{y}$.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*- and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80\ \text{m/s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

Note:
Review of Kinematic Equations (constant a)
Equation:

$$x = x_0 + \bar{v}t$$

Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

Equation:

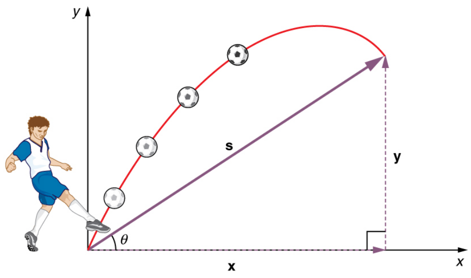
$$v = v_0 + \text{at}$$

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$



The total displacement **s** of a soccer ball at a point along its path. The vector **s** has components **x** and **y** along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A \cos \theta$ and $A_y = A \sin \theta$ are used. The magnitude of the components of displacement **s** along these axes are x and y. The magnitudes of the components of the velocity **v** are $v_x = v \cos \theta$ and $v_y = v \sin \theta$, where v is the magnitude of the velocity and θ is its direction, as shown in [link]. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

Equation:

$$\text{Horizontal Motion}(a_x = 0)$$

Equation:

$$x = x_0 + v_x t$$

Equation:

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

Equation:

$$\text{Vertical Motion}(\text{assuming positive is up } a_y = -g = -9.80 \text{m/s}^2)$$

Equation:

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

Equation:

$$v_y = v_{0y} - \text{gt}$$

Equation:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Equation:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* **s** *and velocity* **v**. Because the *x* - and *y* -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ in the following form, where θ is the direction of the displacement **s** and θ_v is the direction of the velocity **v**:

Total displacement and velocity
Equation:

$$s = \sqrt{x^2 + y^2}$$

Equation:

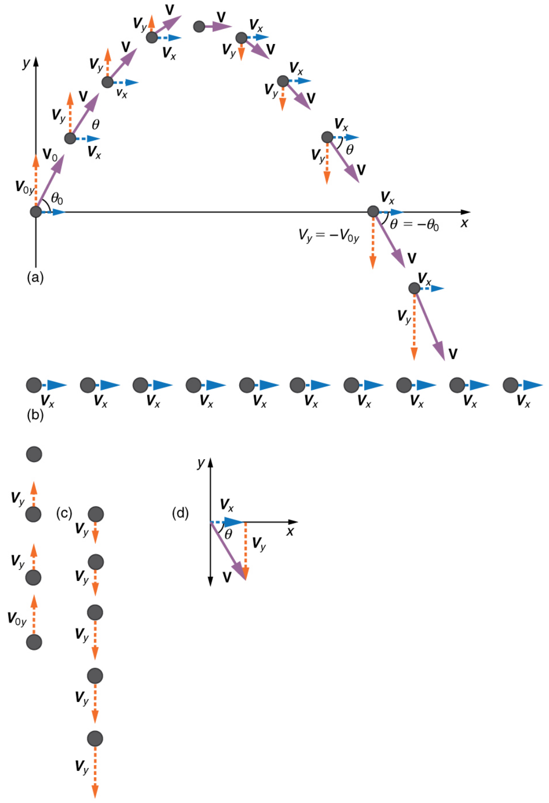
$$\theta = \tan^{-1}(y/x)$$

Equation:

$$v = \sqrt{v_x^2 + v_y^2}$$

Equation:

$$\theta_v = \tan^{-1}(v_y/v_x).$$



(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

Example:

A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in [link]. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0

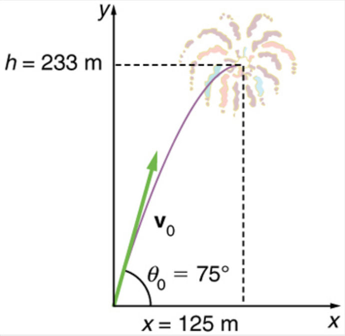
and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By “height” we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

Equation:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$



The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

Equation:

$$0 = v_{0y}^2 - 2\text{gy}.$$

Solving for y gives

Equation:

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, the component of the initial velocity in the y-direction. It is given by $v_{0y} = v_0 \sin \theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0^\text{o}$ is the initial angle. Thus,

Equation:

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75^\text{o}) = 67.6 \text{ m/s}.$$

and y is

Equation:

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

so that

Equation:

$$y = 233\text{m}.$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

Equation:

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

Equation:

$$\begin{aligned} t &= \frac{2y}{(v_{0\text{y}} + v_y)} = \frac{2(233\ \text{m})}{(67.6\ \text{m/s})} \\ &= 6.90\ \text{s}. \end{aligned}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}\text{gt}^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

Equation:

$$x = v_x t,$$

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 \cos\theta_0$. Now,

Equation:

$$v_x = v_0 \cos\theta_0 = (70.0\ \text{m/s})(\cos 75.0^\text{o}) = 18.1\ \text{m/s}.$$

The time t for both motions is the same, and so x is

Equation:

$$x = (18.1\ \text{m/s})(6.90\ \text{s}) = 125\ \text{m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

Equation:

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

Note:

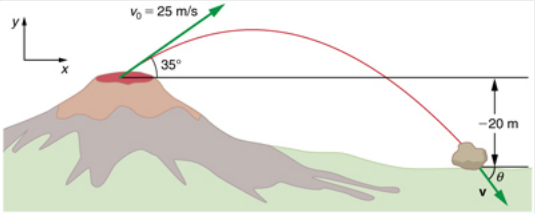
Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Example:

Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in [link]. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?



The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

Equation:

$$y = y_0 + v_{0y}t - \frac{1}{2}\text{gt}^2.$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0\text{ m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin \theta_0$ = (25.0 m/s)(sin 35.0°) = 14.3 m/s. Substituting known values yields

Equation:

$$-20.0\text{ m} = (14.3\text{ m/s})t - \left(4.90\text{ m/s}^2\right)t^2.$$

Rearranging terms gives a quadratic equation in t:

Equation:

$$\left(4.90\text{ m/s}^2\right)t^2 - (14.3\text{ m/s})t - (20.0\text{ m}) = 0.$$

This expression is a quadratic equation of the form $\text{at}^2 + \text{bt} + c = 0$, where the constants are $a = 4.90$, $b =- 14.3$, and $c =- 20.0$. Its solutions are given by the quadratic formula:

Equation:

$$t = \frac{-b \pm \sqrt{b^2 - 4\text{ac}}}{2a}.$$

This equation yields two solutions: $t = 3.96$ and $t =- 1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96$ s or -1.03 s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

Equation:

$$t = 3.96\text{ s}.$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

Equation:

$$v_x = v_0 \cos \theta_0 = (25.0\text{ m/s})(\cos 35^\text{o}) = 20.5\text{ m/s}.$$

The final vertical velocity is given by the following equation:

Equation:

$$v_y = v_{0y} - \text{gt},$$

where $v_{0\text{y}}$ was found in part (a) to be 14.3 m/s. Thus,

Equation:

$$v_y = 14.3\text{ m/s} - (9.80\text{ m/s}^2)(3.96\text{ s})$$

so that

Equation:

$$v_y = -24.5\text{ m/s}.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

Equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5\ \text{m/s})^2 + (-24.5\ \text{m/s})^2},$$

which gives

Equation:

$$v = 31.9\ \text{m/s}.$$

The direction θ_v is found from the equation:

Equation:

$$\theta_v = \tan^{-1}(v_y/v_x)$$

so that

Equation:

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

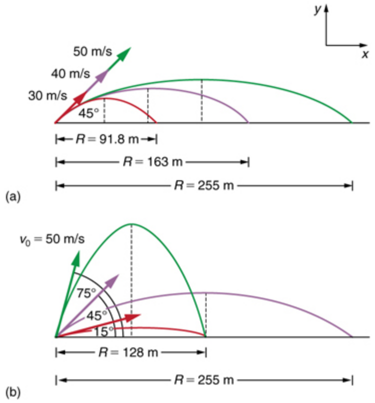
Equation:

$$\theta_v = -50.1^\text{o}.$$

Discussion for (b)

The negative angle means that the velocity is 50.1^o below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See [link].)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.



Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in [link](a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in [link](b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^\circ$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38°. Interestingly, for every initial angle except 45°, there are two angles that give the same range—the sum of those angles is 90°. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

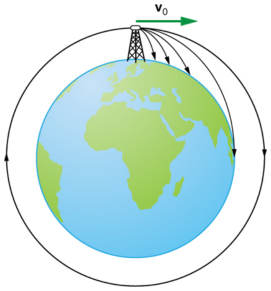
Equation:

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See [link].) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

Note:
PhET Explorations: Projectile Motion
Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.
https://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\mathbf{s}$ are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v \cos \theta$ and $v_y = v \sin \theta$, where v is the magnitude of the velocity and θ is its direction.

Analyze the motion of the projectile in the horizontal direction using the following equations:

Equation:

$$\text{Horizontal motion}(a_x = 0)$$

Equation:

$$x = x_0 + v_x t$$

Equation:

$$v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$$

Analyze the motion of the projectile in the vertical

Equation:

$$\text{Vertical motion}(\text{Assuming positive direction is up; } a_y = -g = -9.80\ \text{m/s}^2)$$

Equation:

$$y = y_0 + \frac{1}{2}(v_{0y}$$

direction using the following equations:

Recombine the horizontal and vertical components of location and/or velocity using the following equations:

Equation:

$$s=\sqrt{x^2+y^2}$$

Equation:

$$\theta=\tan^{-1}(y/x)$$

Equation:

$$v=\sqrt{v_x^2+v_y^2}$$

Equation:

$$\theta_{\text{v}}=\tan^{-1}(v_y/v_x).$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by
  Equation:

$$h=\frac{v_{0y}^2}{2g}.$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by
  Equation:

$$R=\frac{v_0^2\sin 2\theta_0}{g}.$$

Conceptual Questions

Exercise:

Problem:

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t=0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t=0$?

Exercise:

Problem:

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

Exercise:

Problem:

For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

Exercise:

Problem:

During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

Problems & Exercises

Exercise:

Problem:

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0^{o} above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

Solution:

$$\begin{array}{lll}x & = & 1.30\text{ m}\times 10^{2}\\ y & = & 30.9\text{ m.}\end{array}$$

Exercise:

Problem:

A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

Exercise:

Problem:

A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

Solution:

(a) 3.50 s

(b) 28.6 m/s (c) 34.3 m/s

(d) 44.7 m/s, 50.2^{o} below horizontal

Exercise:

Problem:

(a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32^{o} ramp at a speed of 40.0 m/s (144 km/h). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

Exercise:

Problem:

An archer shoots an arrow at a 75.0 m distant target; the bull’s-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull’s-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

Solution:

(a) 18.4°

(b) The arrow will go over the branch.

Exercise:

Problem:

A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

Exercise:

Problem: Verify the ranges for the projectiles in [link](a) for $\theta = 45^{\circ}$ and the given initial velocities.

Solution:

$$R = \frac{v_0^2}{\text{sin}2\theta_0 g}$$

For $\theta = 45^{\circ}, \quad R = \frac{v_0^2}{g}$

$R = 91.8\text{ m}$ for $v_0 = 30\text{ m/s}$; $R = 163\text{ m}$ for $v_0 = 40\text{ m/s}$; $R = 255\text{ m}$ for $v_0 = 50\text{ m/s}$.

Exercise:

Problem:

Verify the ranges shown for the projectiles in [link](b) for an initial velocity of 50 m/s at the given initial angles.

Exercise:

Problem:

The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3\text{ km}$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

Solution:

(a) 560 m/s

(b) $8.00 \times 10^3\text{ m}$

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

Exercise:

Problem:

An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of $60^{\rm o}$ above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

Exercise:

Problem:

In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

Solution:

1.50 m, assuming launch angle of $45^{\rm o}$

Exercise:

Problem:

The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

Exercise:

Problem:

Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

Solution:

$\theta = 6.1^{\rm o}$

yes, the ball lands at 5.3 m from the net

Exercise:

Problem:

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of $25^{\rm o}$ relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

Exercise:

Problem:

Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

Solution:

(a) -0.486 m

(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

Exercise:

Problem:

An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

Exercise:

Problem:

An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0° below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

Solution:

4.23 m. No, the owl is not lucky; he misses the nest.

Exercise:

Problem:

Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be 40° above the horizontal.

Exercise:

Problem:

Can a goalkeeper at her/ his goal kick a soccer ball into the opponent’s goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

Solution:

No, the maximum range (neglecting air resistance) is about 92 m.

Exercise:

Problem:

The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

Exercise:

Problem:

In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)

Solution:

15.0 m/s

Exercise:

Problem:

A basketball player is running at $5.00\ \text{m/s}$ directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

Exercise:

Problem:

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

Solution:

(a) 24.2 m/s

(b) The ball travels a total of 57.4 m with the brief gust of wind.

Exercise:

Problem:

Prove that the trajectory of a projectile is parabolic, having the form $y = \text{ax} + \text{bx}^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t – (1/2)\text{gt}^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = \text{ax} + \text{bx}^2$ where a and b are constants.

Exercise:

Problem:

Derive $R = \frac{v_0^2 \sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$

Solution:

$y - y_0 = 0 = v_{0y}t - \frac{1}{2}\text{gt}^2 = (v_0 \sin \theta)t - \frac{1}{2}\text{gt}^2$,

so that $t = \frac{2(v_0 \sin \theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \cos \theta)t = R$, and substituting for t gives:

$$R = v_0 \cos\theta\left(\frac{2v_0 \sin\theta}{g}\right) = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

since $2 \sin\theta \cos\theta = \sin 2\theta$, the range is:

$$R = \frac{{v_0}^2 \sin 2\theta}{g}.$$

Exercise:

Problem:

Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

Exercise:

Problem:

Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

Glossary

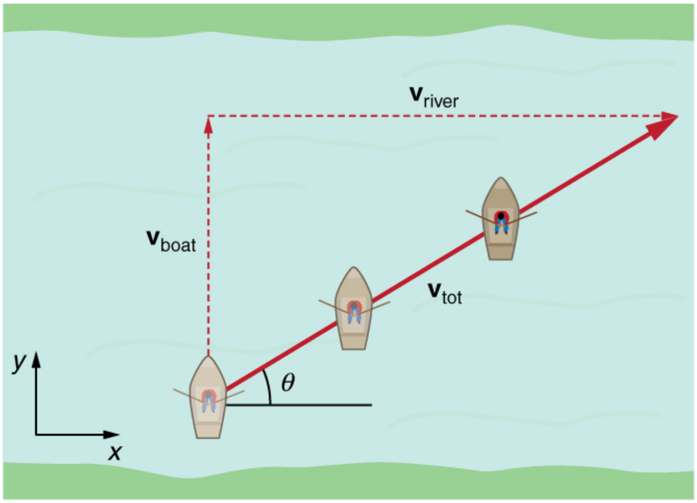
air resistance	a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero
kinematics	the study of motion without regard to mass or force
motion	displacement of an object as a function of time
projectile	an object that travels through the air and experiences only acceleration due to gravity
projectile motion	the motion of an object that is subject only to the acceleration of gravity
range	the maximum horizontal distance that a projectile travels
trajectory	the path of a projectile through the air

Addition of Velocities

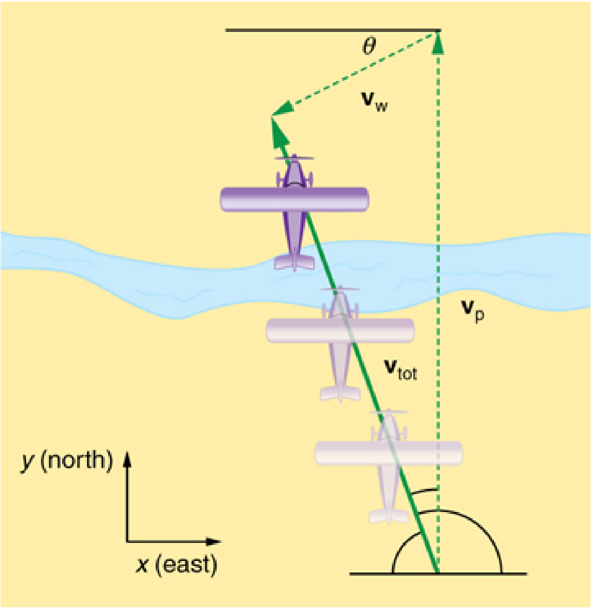
- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in [link]. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in [link]. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.


$\mathbf{v}_{\text{river}}$
$\mathbf{v}_{\text{boat}}$
$\mathbf{v}_{\text{tot}}$
θ
y
x

A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.



An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in [link] and [link]. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*- and *y*-axes of an appropriately chosen coordinate system:

Equation:

$$v_x = v\text{ cos }\theta$$

Equation:

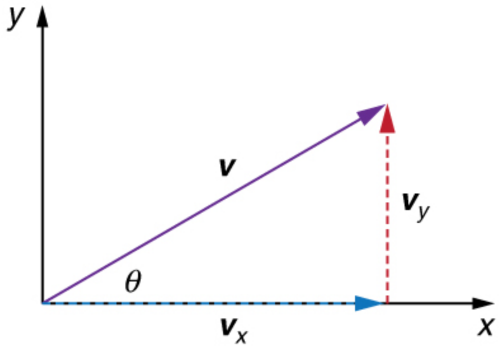
$$v_y = v\text{ sin }\theta$$

Equation:

$$v = \sqrt{v_x^2 + v_y^2}$$

Equation:

$$\theta = \text{tan}^{-1}(v_y/v_x).$$



The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

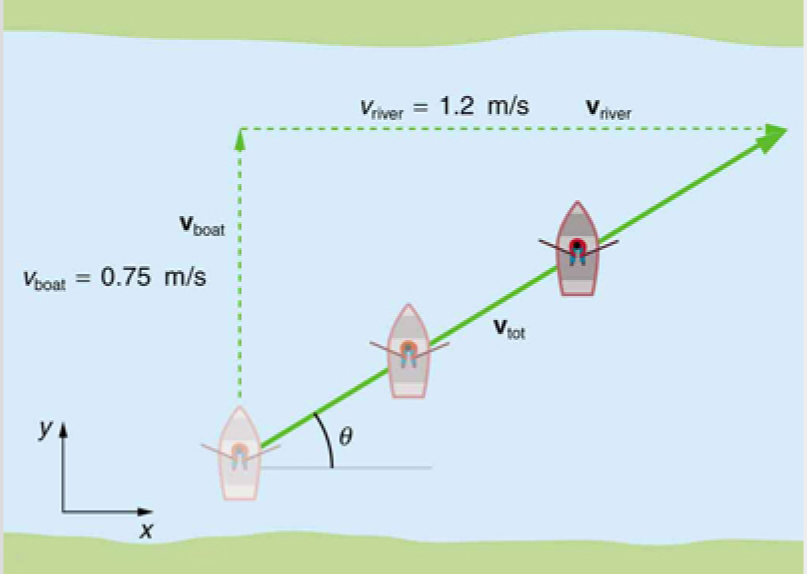
Note:

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example:

Adding Velocities: A Boat on a River


v_{river} = 1.2 m/s
$\mathbf{v}_{\text{river}}$
$\mathbf{v}_{\text{boat}}$
v_{boat} = 0.75 m/s
$\mathbf{v}_{\text{tot}}$
θ
y
x

A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to [link], which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in [link]. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

Equation:

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2},$$

where

Equation:

$$v_x = v_{\text{river}} = 1.20 \text{ m/s}$$

and

Equation:

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}.$$

Thus,

Equation:

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2}$$

yielding

Equation:

$$v_{\text{tot}} = 1.42 \text{ m/s}.$$

The direction of the total velocity θ is given by:

Equation:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.750/1.20).$$

This equation gives

Equation:

$$\theta = 32.0^{\text{o}}.$$

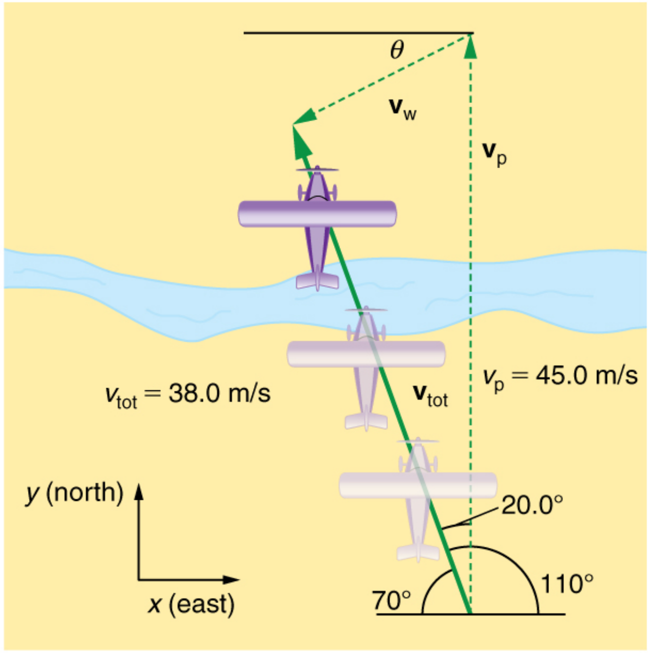
Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with [link]. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0º) the total velocity has relative to the riverbank.

Example:

Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in [link]. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0º west of north.


θ
$\mathbf{v}_w$
$\mathbf{v}_p$
v_{tot} = 38.0 m/s
$\mathbf{v}_{tot}$
v_p = 45.0 m/s
y (north)
x (east)
20.0°
70°
110°

An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in [link], we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{\text{p}x} = 0$ and $v_{\text{p}y} = v_{\text{p}}$. That is,

Equation:

$$v_{\text{tot}x} = v_{\text{w}x}$$

and

Equation:

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}.$$

We can use the first of these two equations to find $v_{\text{w}x}$:

Equation:

$$v_{\text{w}x} = v_{\text{tot}x} = v_{\text{tot}}\text{cos } 110^{\text{o}}.$$

Because $v_{\text{tot}} = 38.0 \text{ m/s}$ and cos $110^{\text{o}} = - 0.342$ we have

Equation:

$$v_{\text{w}x} = (38.0 \text{ m/s})(–0.342) = –13 \text{ m/s}.$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find $v_{\text{w}y}$ we note that

Equation:

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}$$

Here $v_{\text{tot}y} = v_{\text{tot}}\text{sin }110^{\text{o}}$; thus,

Equation:

$$v_{\text{w}y} = (38.0\text{ m/s})(0.940) - 45.0\text{ m/s} = -9.29\text{ m/s}.$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity $v_{\text{w}x}$ and $v_{\text{w}y}$ are known, we can find the magnitude and direction of $\mathbf{v}_{\text{w}}$. First, the magnitude is

Equation:

$$\begin{array}{lll} v_{\text{w}} & = & \sqrt{v_{\text{w}x}^{2} + v_{\text{w}y}^{2}} \\ & = & \sqrt{(-13.0\text{ m/s})^{2} + (-9.29\text{ m/s})^{2}} \end{array}$$

so that

Equation:

$$v_{\text{w}} = 16.0\text{ m/s}.$$

The direction is:

Equation:

$$\theta = \text{tan}^{-1}(v_{\text{w}y}/v_{\text{w}x}) = \text{tan}^{-1}(-9.29/-13.0)$$

giving

Equation:

$$\theta = 35.6^{\text{o}}.$$

Discussion

The wind’s speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in [link]. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

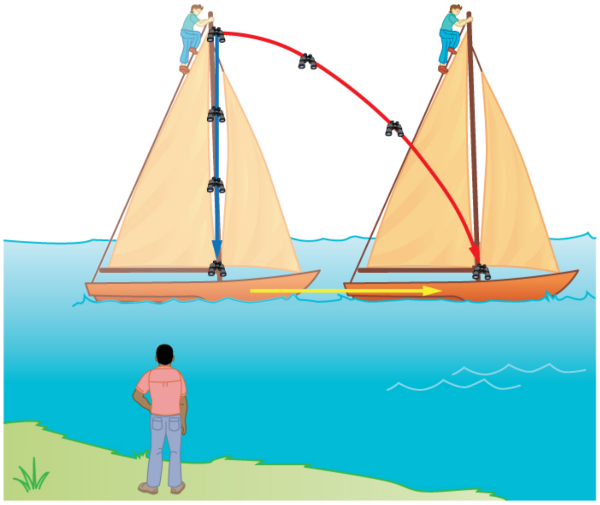
Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3,000\text{ km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See [link].) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in [link]. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

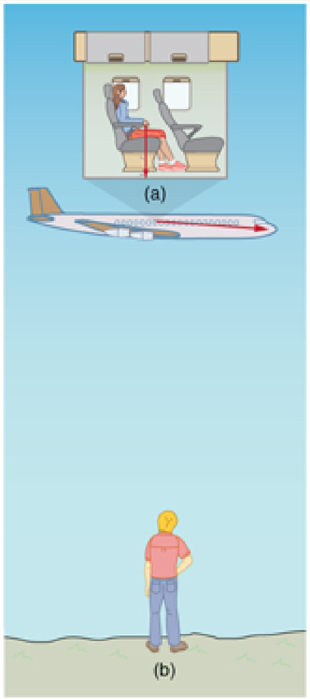


Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example:

Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?



The motion of

a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

Equation:

$$v_y{}^2 = v_{0y}{}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

Equation:

$$ {v_y}^2 = 0^2 - 2(9.80\text{ m/s}^2)(-1.50\text{ m} - 0\text{ m}) = 29.4\text{ m}^2/\text{s}^2 $$

yielding

Equation:

$$ v_y = -5.42\text{ m/s}. $$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\text{ m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260\text{ m/s}$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

Equation:

$$ v = \sqrt{{v_x}^2 + {v_y}^2}. $$

Thus,

Equation:

$$ v = \sqrt{(260\text{ m/s})^2 + (-5.42\text{ m/s})^2} $$

yielding

Equation:

$$ v = 260.06\text{ m/s}. $$

The direction is given by:

Equation:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260)$$

so that

Equation:

$$\theta = \tan^{-1}(-0.0208) = -1.19^{\text{o}}.$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 – 5.42)\ \text{m/s}$; rather, it is $260.06\ \text{m/s}$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See [link].) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Note:

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

Note:
PhET Explorations: Motion in 2D
Try the new "Motion in 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Motion in 2D

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as
  Equation:
  $$v_x = v \cos \theta$$
  Equation:
  $$v_y = v \sin \theta$$
  Equation:

$$v=\sqrt{v_x^2+v_y^2}$$

Equation:

$$\theta=\tan^{-1}(v_y/v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

Exercise:

Problem:

What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

Exercise:

Problem:

A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn’t he need to keep his eyes on the ball?

Exercise:

Problem:

If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

Exercise:

Problem:

The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

Exercise:

Problem:

A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

Exercise:

Problem:

Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45º south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

Solution:

(a) 35.8 km, 45º south of east

(b) 5.53 m/s, 45º south of east

(c) 56.1 km, 45° south of east

Exercise:

Problem:

A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

Exercise:

Problem:

Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

Solution:

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

Exercise:

Problem:

Verify that the coin dropped by the airline passenger in the [link] travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

Exercise:

Problem:

A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

Solution:

$17.0\text{ m/s}, 22.1^\circ$

Exercise:

Problem:

A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

Exercise:

Problem:

(a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

Solution:

(a) $230\text{ m/s}, 8.0^\circ$ south of west

(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

Exercise:

Problem:

(a) In what direction would the ship in [link] have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s? (b) What would its speed be relative to the Earth?

Exercise:

Problem:

(a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in [link]). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

Solution:

(a) 63.5 m/s

(b) 29.6 m/s

Exercise:

Problem:

A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

Exercise:

Problem:

The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

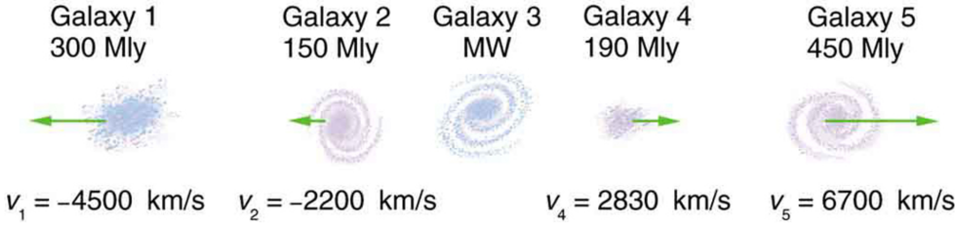
Solution:

6.68 m/s, 53.3° south of west

Exercise:

Problem:

The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. [link] illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.



Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

Exercise:

Problem:

(a) Use the distance and velocity data in [link] to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

Solution:

(a) $H_{\text{average}} = 14.9 \frac{\text{km/s}}{\text{Mly}}$

(b) 20.2 billion years

Exercise:

Problem:

An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

Exercise:

Problem:

A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00° west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

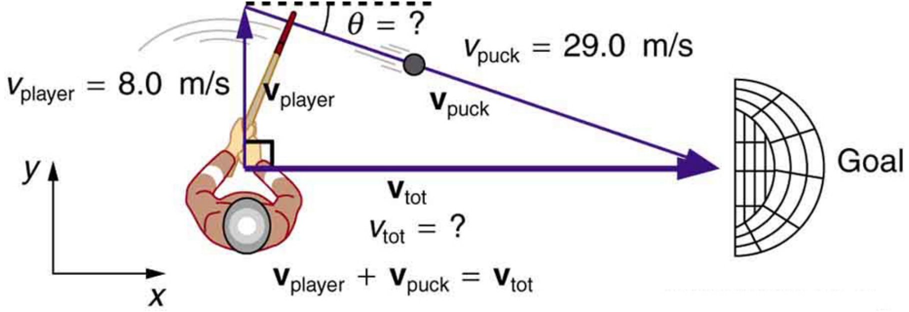
Solution:

1.72 m/s, 42.3° north of east

Exercise:

Problem:

An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in [link]. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?


θ = ?
$v_{\text{puck}} = 29.0$ m/s
$v_{\text{player}} = 8.0$ m/s
$\mathbf{v}_{\text{player}}$
$\mathbf{v}_{\text{puck}}$
Goal
y
x
$\mathbf{v}_{\text{tot}}$
$v_{\text{tot}} = ?$
$\mathbf{v}_{\text{player}} + \mathbf{v}_{\text{puck}} = \mathbf{v}_{\text{tot}}$

An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

Exercise:

Problem:

Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

Exercise:

Problem:

Unreasonable Results A commercial airplane has an air speed of $280\ \text{m/s}$ due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

Exercise:

Problem:

Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Glossary

classical relativity

the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

relative velocity
the velocity of an object as observed from a particular reference frame

relativity
the study of how different observers moving relative to each other measure the same phenomenon

velocity
speed in a given direction

vector addition
the rules that apply to adding vectors together

Introduction to Thermodynamics class="introduction"

A steam engine uses heat transfer to do work. Tourists regularly ride this narrow-gauge steam engine train near the San Juan Skyway in Durango, Colorado, part of the National Scenic Byways Program. (credit: Dennis Adams)

Heat transfer is energy in transit, and it can be used to do work. It can also be converted to any other form of energy. A car engine, for example, burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance, converting its energy into a variety of other forms—into the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the heat transfer produced from burning fuel in the engine does not do work on the gas. Rather, the energy is released into the environment, implying that the engine is quite inefficient.

It is often said that modern gasoline engines cannot be made to be significantly more efficient. We hear the same about heat transfer to electrical energy in large power stations, whether they are coal, oil, natural gas, or nuclear powered. Why is that the case? Is the inefficiency caused by design problems that could be solved with better engineering and superior materials? Is it part of some money-making conspiracy by those who sell energy? Actually, the truth is more interesting, and reveals much about the nature of heat transfer.

Basic physical laws govern how heat transfer for doing work takes place and place insurmountable limits onto its efficiency. This chapter will explore these laws as well as many applications and concepts associated with them. These topics are part of *thermodynamics*—the study of heat transfer and its relationship to doing work.

The First Law of Thermodynamics

- Define the first law of thermodynamics.
- Describe how conservation of energy relates to the first law of thermodynamics.
- Identify instances of the first law of thermodynamics working in everyday situations, including biological metabolism.
- Calculate changes in the internal energy of a system, after accounting for heat transfer and work done.

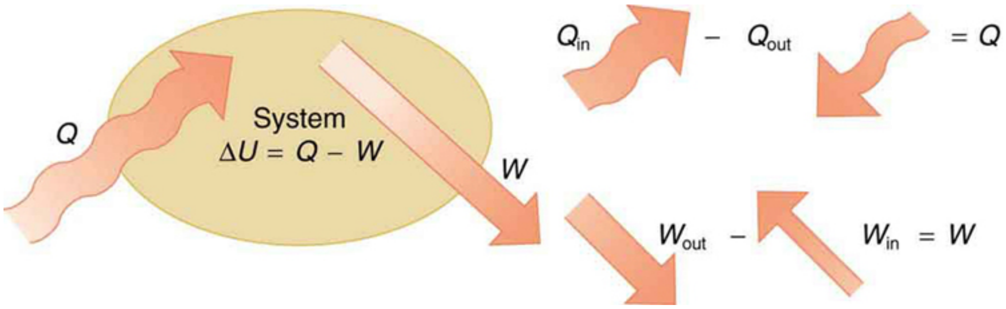
This boiling tea kettle represents energy in motion. The water in the kettle is turning to water vapor because heat is being transferred from the stove to the kettle. As the entire system gets hotter, work is done—from the evaporation of the water to the whistling of the kettle. (credit: Gina Hamilton)

If we are interested in how heat transfer is converted into doing work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. The **first law of thermodynamics** states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

Equation:

$$\Delta U = Q - W.$$

Here ΔU is the *change in internal energy* U of the system. Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfer into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work. (See [link].) We will now examine Q, W, and ΔU further.



The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the system. Q is positive for net heat transfer *into* the system. W is the total work done on and by the system. W is positive when more work is done *by* the system than on it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

Note:
Making Connections: Law of Thermodynamics and Law of Conservation of Energy

The first law of thermodynamics is actually the law of conservation of energy stated in a form most useful in thermodynamics. The first law gives the relationship between heat transfer, work done, and the change in internal energy of a system.

Heat *Q* and Work *W*

Heat transfer (Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system. The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process, involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results.For example, both can cause a temperature increase. Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work. This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy U of a system. Internal energy is a form of energy completely different from either heat or work.

Internal Energy *U*

We can think about the internal energy of a system in two different but consistent ways. The first is the atomic and molecular view, which examines the system on the atomic and molecular scale. The **internal energy** U of a system is the sum of the kinetic and potential energies of its atoms and molecules. Recall that kinetic plus potential energy is called mechanical energy. Thus internal energy is the sum of atomic and molecular mechanical energy. Because it is impossible to keep track of all individual atoms and molecules, we must deal with averages and distributions. A second way to view the internal energy of a system is in terms of its macroscopic characteristics, which are very similar to atomic and molecular average values.

Macroscopically, we define the change in internal energy ΔU to be that given by the first law of thermodynamics:

Equation:

$$\Delta U = Q - W.$$

Many detailed experiments have verified that $\Delta U = Q - W$, where ΔU is the change in total kinetic and potential energy of all atoms and molecules in a system. It has also been determined experimentally that the internal energy U of a system depends only on the state of the system and *not how it reached that state*. More specifically, U is found to be a function of a few macroscopic quantities (pressure, volume, and temperature, for example), independent of past history such as whether there has been heat transfer or work done. This independence means that if we know the state of a system, we can calculate changes in its internal energy U from a few macroscopic variables.

Note:

Making Connections: Macroscopic and Microscopic

In thermodynamics, we often use the macroscopic picture when making calculations of how a system behaves, while the atomic and molecular picture gives underlying explanations in terms of averages and distributions. We shall see this again in later sections of this chapter. For example, in the topic of entropy, calculations will be made using the atomic and molecular view.

To get a better idea of how to think about the internal energy of a system, let us examine a system going from State 1 to State 2. The system has internal energy U_1 in State 1, and it has internal energy U_2 in State 2, no matter how it got to either state. So the change in internal energy $\Delta U = U_2 - U_1$ is independent of what caused the change. In other words, ΔU *is independent of path*. By path, we mean the method of getting from the starting point to the ending point. Why is this independence important? Note that $\Delta U = Q - W$. Both Q and W *depend on path*, but ΔU does not. This path independence means that internal energy U is easier to consider than either heat transfer or work done.

Example:

Calculating Change in Internal Energy: The Same Change in U is Produced by Two Different Processes

(a) Suppose there is heat transfer of 40.00 J to a system, while the system does 10.00 J of work. Later, there is heat transfer of 25.00 J out of the system while 4.00 J of work is done on the system. What is the net change in internal energy of the system?

(b) What is the change in internal energy of a system when a total of 150.00 J of heat transfer occurs out of (from) the system and 159.00 J of work is done on the system? (See [link]).

Strategy

In part (a), we must first find the net heat transfer and net work done from the given information. Then the first law of thermodynamics

$(\Delta U = Q - W)$ can be used to find the change in internal energy. In part (b), the net heat transfer and work done are given, so the equation can be used directly.

Solution for (a)

The net heat transfer is the heat transfer into the system minus the heat transfer out of the system, or

Equation:

$$Q = 40.00\text{ J} - 25.00\text{ J} = 15.00\text{ J}.$$

Similarly, the total work is the work done by the system minus the work done on the system, or

Equation:

$$W = 10.00\text{ J} - 4.00\text{ J} = 6.00\text{ J}.$$

Thus the change in internal energy is given by the first law of thermodynamics:

Equation:

$$\Delta U = Q - W = 15.00\text{ J} - 6.00\text{ J} = 9.00\text{ J}.$$

We can also find the change in internal energy for each of the two steps. First, consider 40.00 J of heat transfer in and 10.00 J of work out, or

Equation:

$$\Delta U_1 = Q_1 - W_1 = 40.00\text{ J} - 10.00\text{ J} = 30.00\text{ J}.$$

Now consider 25.00 J of heat transfer out and 4.00 J of work in, or

Equation:

$$\Delta U_2 = Q_2 - W_2 = -25.00\text{ J} - (-4.00\text{ J}) = -21.00\text{ J}.$$

The total change is the sum of these two steps, or

Equation:

$$\Delta U = \Delta U_1 + \Delta U_2 = 30.00\text{ J} + (-21.00\text{ J}) = 9.00\text{ J}.$$

Discussion on (a)

No matter whether you look at the overall process or break it into steps, the change in internal energy is the same.

Solution for (b)

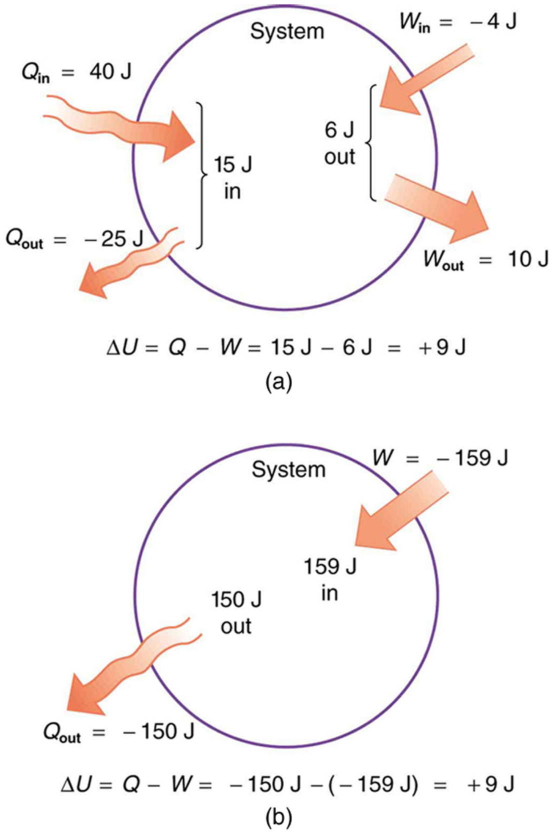
Here the net heat transfer and total work are given directly to be $Q = -150.00\text{ J}$ and $W = -159.00\text{ J}$, so that

Equation:

$$\Delta U = Q - W = -150.00\text{ J} - (-159.00\text{ J}) = 9.00\text{ J}.$$

Discussion on (b)

A very different process in part (b) produces the same 9.00-J change in internal energy as in part (a). Note that the change in the system in both parts is related to ΔU and not to the individual Qs or Ws involved. The system ends up in the *same* state in both (a) and (b). Parts (a) and (b) present two different paths for the system to follow between the same starting and ending points, and the change in internal energy for each is the same—it is independent of path.


System
Q_{in} = 40 J
W_{in} = −4 J
15 J in
6 J out
Q_{out} = −25 J
W_{out} = 10 J
$\Delta U = Q - W = 15\text{ J} - 6\text{ J} = +9\text{ J}$
(a)
System
W = −159 J
159 J in
150 J out
Q_{out} = −150 J
$\Delta U = Q - W = -150\text{ J} - (-159\text{ J}) = +9\text{ J}$
(b)

Two different processes produce the same change in a system. (a) A total of 15.00 J of heat transfer occurs into the system, while work takes out a total of 6.00 J. The change in internal energy is $\Delta U = Q - W = 9.00\text{ J}$. (b) Heat transfer removes 150.00 J from the system while work puts 159.00 J into it, producing an increase of 9.00 J in internal energy. If the system starts out in the same state in (a) and (b), it will end up in the same final state in either case—its final state is related to internal

energy, not how that energy
was acquired.

Human Metabolism and the First Law of Thermodynamics

Human metabolism is the conversion of food into heat transfer, work, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. We now take another look at these topics via the first law of thermodynamics. Considering the body as the system of interest, we can use the first law to examine heat transfer, doing work, and internal energy in activities ranging from sleep to heavy exercise. What are some of the major characteristics of heat transfer, doing work, and energy in the body? For one, body temperature is normally kept constant by heat transfer to the surroundings. This means Q is negative. Another fact is that the body usually does work on the outside world. This means W is positive. In such situations, then, the body loses internal energy, since $\Delta U = Q - W$ is negative.

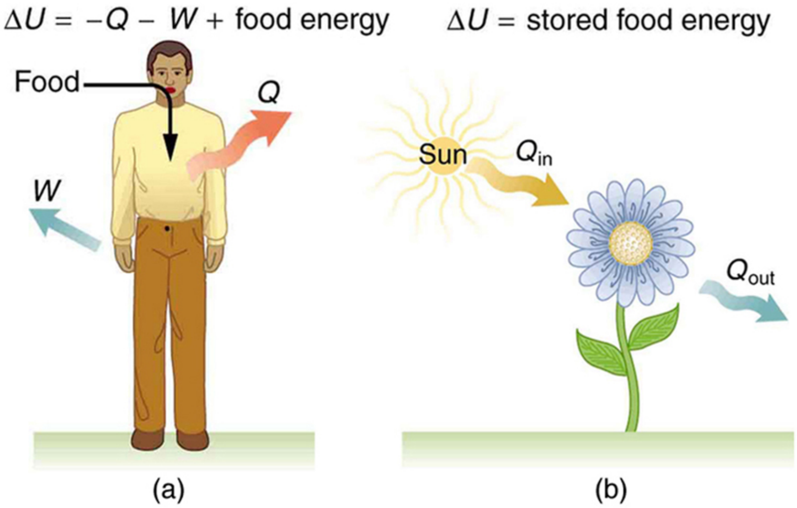
Now consider the effects of eating. Eating increases the internal energy of the body by adding chemical potential energy (this is an unromantic view of a good steak). The body *metabolizes* all the food we consume. Basically, metabolism is an oxidation process in which the chemical potential energy of food is released. This implies that food input is in the form of work. Food energy is reported in a special unit, known as the Calorie. This energy is measured by burning food in a calorimeter, which is how the units are determined.

In chemistry and biochemistry, one calorie (spelled with a *lowercase* c) is defined as the energy (or heat transfer) required to raise the temperature of one gram of pure water by one degree Celsius. Nutritionists and weight-watchers tend to use the *dietary* calorie, which is frequently called a Calorie (spelled with a *capital* C). One food Calorie is the energy needed to raise the temperature of one *kilogram* of water by one degree Celsius. This means that one dietary Calorie is equal to one kilocalorie for the chemist, and one must be careful to avoid confusion between the two.

Again, consider the internal energy the body has lost. There are three places this internal energy can go—to heat transfer, to doing work, and to stored fat (a tiny fraction also goes to cell repair and growth). Heat transfer and doing work take internal energy out of the body, and food puts it back. If you eat just the right amount of food, then your average internal energy remains constant. Whatever you lose to heat transfer and doing work is replaced by food, so that, in the long run, $\Delta U = 0$. If you overeat repeatedly, then ΔU is always positive, and your body stores this extra internal energy as fat. The reverse is true if you eat too little. If ΔU is negative for a few days, then the body metabolizes its own fat to maintain body temperature and do work that takes energy from the body. This process is how dieting produces weight loss.

Life is not always this simple, as any dieter knows. The body stores fat or metabolizes it only if energy intake changes for a period of several days. Once you have been on a major diet, the next one is less successful because your body alters the way it responds to low energy intake. Your basal metabolic rate (BMR) is the rate at which food is converted into heat transfer and work done while the body is at complete rest. The body adjusts its basal metabolic rate to partially compensate for over-eating or under-eating. The body will decrease the metabolic rate rather than eliminate its own fat to replace lost food intake. You will chill more easily and feel less energetic as a result of the lower metabolic rate, and you will not lose weight as fast as before. Exercise helps to lose weight, because it produces both heat transfer from your body and work, and raises your metabolic rate even when you are at rest. Weight loss is also aided by the quite low efficiency of the body in converting internal energy to work, so that the loss of internal energy resulting from doing work is much greater than the work done.It should be noted, however, that living systems are not in thermalequilibrium.

The body provides us with an excellent indication that many thermodynamic processes are *irreversible*. An irreversible process can go in one direction but not the reverse, under a given set of conditions. For example, although body fat can be converted to do work and produce heat transfer, work done on the body and heat transfer into it cannot be converted to body fat. Otherwise, we could skip lunch by sunning ourselves or by walking down stairs. Another example of an irreversible thermodynamic process is photosynthesis. This process is the intake of one form of energy—light—by plants and its conversion to chemical potential energy. Both applications of the first law of thermodynamics are illustrated in [link]. One great advantage of conservation laws such as the first law of thermodynamics is that they accurately describe the beginning and ending points of complex processes, such as metabolism and photosynthesis, without regard to the complications in between. [link] presents a summary of terms relevant to the first law of thermodynamics.


$\Delta U = -Q - W +$ food energy
$\Delta U =$ stored food energy
Food
Q
W
Sun
Q_{in}
Q_{out}
(a)
(b)

(a) The first law of thermodynamics applied to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, while food intake replaces it. (Food intake may be considered as work done on the body.) (b) Plants convert part of the radiant heat transfer in sunlight to stored chemical energy, a process called photosynthesis.

Term	Definition
U	Internal energy—the sum of the kinetic and potential energies of a system’s atoms and molecules. Can be divided into many subcategories, such as thermal and chemical energy. Depends only on the state of a system (such as its P, V, and T), not on how the energy entered the system. Change in internal energy is path independent.
Q	Heat—energy transferred because of a temperature difference. Characterized by random molecular motion. Highly dependent on path. Q entering a system is positive.
W	Work—energy transferred by a force moving through a distance. An organized, orderly process. Path dependent. W done by a system (either against an external force or to increase the volume of the system) is positive.

Summary of Terms for the First Law of Thermodynamics, $\Delta U = Q - W$

Section Summary

- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net heat transfer (the sum of all heat transfer into and out of the system), and W is the net work done (the sum of all work done on or by the system).
- Both Q and W are energy in transit; only ΔU represents an independent quantity capable of being stored.

- The internal energy U of a system depends only on the state of the system and not how it reached that state.
- Metabolism of living organisms, and photosynthesis of plants, are specialized types of heat transfer, doing work, and internal energy of systems.

Conceptual Questions

Exercise:

Problem:

Describe the photo of the tea kettle at the beginning of this section in terms of heat transfer, work done, and internal energy. How is heat being transferred? What is the work done and what is doing it? How does the kettle maintain its internal energy?

Exercise:

Problem:

The first law of thermodynamics and the conservation of energy, as discussed in Conservation of Energy, are clearly related. How do they differ in the types of energy considered?

Exercise:

Problem:

Heat transfer Q and work done W are always energy in transit, whereas internal energy U is energy stored in a system. Give an example of each type of energy, and state specifically how it is either in transit or resides in a system.

Exercise:

Problem:

How do heat transfer and internal energy differ? In particular, which can be stored as such in a system and which cannot?

Exercise:

Problem:

If you run down some stairs and stop, what happens to your kinetic energy and your initial gravitational potential energy?

Exercise:

Problem:

Give an explanation of how food energy (calories) can be viewed as molecular potential energy (consistent with the atomic and molecular definition of internal energy).

Exercise:

Problem:

Identify the type of energy transferred to your body in each of the following as either internal energy, heat transfer, or doing work: (a) basking in sunlight; (b) eating food; (c) riding an elevator to a higher floor.

Problems & Exercises

Exercise:

Problem:

What is the change in internal energy of a car if you put 12.0 gal of gasoline into its tank? The energy content of gasoline is $1.3 \times 10^8 \text{ J/gal}$. All other factors, such as the car's temperature, are constant.

Solution:

$1.6 \times 10^9 \text{ J}$

Exercise:

Problem:

How much heat transfer occurs from a system, if its internal energy decreased by 150 J while it was doing 30.0 J of work?

Exercise:

Problem:

A system does $1.80 \times 10^8 \text{ J}$ of work while $7.50 \times 10^8 \text{ J}$ of heat transfer occurs to the environment. What is the change in internal energy of the system assuming no other changes (such as in temperature or by the addition of fuel)?

Solution:

$-9.30 \times 10^8 \text{ J}$

Exercise:

Problem:

What is the change in internal energy of a system which does $4.50 \times 10^5 \text{ J}$ of work while $3.00 \times 10^6 \text{ J}$ of heat transfer occurs into the system, and $8.00 \times 10^6 \text{ J}$ of heat transfer occurs to the environment?

Exercise:

Problem:

Suppose a woman does 500 J of work and 9500 J of heat transfer occurs into the environment in the process. (a) What is the decrease in her internal energy, assuming no change in temperature or consumption of food? (That is, there is no other energy transfer.) (b) What is her efficiency?

Solution:

(a) $-1.0 \times 10^4 \text{ J}$, or -2.39 kcal

(b) 5.00%

Exercise:

Problem:

(a) How much food energy will a man metabolize in the process of doing 35.0 kJ of work with an efficiency of 5.00%? (b) How much heat transfer occurs to the environment to keep his temperature constant? Explicitly show how you follow the steps in the Problem-Solving Strategy for thermodynamics found in Problem-Solving Strategies for Thermodynamics.

Exercise:

Problem:

(a) What is the average metabolic rate in watts of a man who metabolizes 10,500 kJ of food energy in one day? (b) What is the maximum amount of work in joules he can do without breaking down fat, assuming a maximum efficiency of 20.0%? (c) Compare his work output with the daily output of a 187-W (0.250-horsepower) motor.

Solution:

(a) 122 W

(b) 2.10×10^6 J

(c) Work done by the motor is 1.61×10^7 J ;thus the motor produces 7.67 times the work done by the man

Exercise:

Problem:

(a) How long will the energy in a 1470-kJ (350-kcal) cup of yogurt last in a woman doing work at the rate of 150 W with an efficiency of 20.0% (such as in leisurely climbing stairs)? (b) Does the time found in part (a) imply that it is easy to consume more food energy than you can reasonably expect to work off with exercise?

Exercise:

Problem:

(a) A woman climbing the Washington Monument metabolizes 6.00×10^2 kJ of food energy. If her efficiency is 18.0%, how much heat transfer occurs to the environment to keep her temperature constant? (b) Discuss the amount of heat transfer found in (a). Is it consistent with the fact that you quickly warm up when exercising?

Solution:

(a) 492 kJ

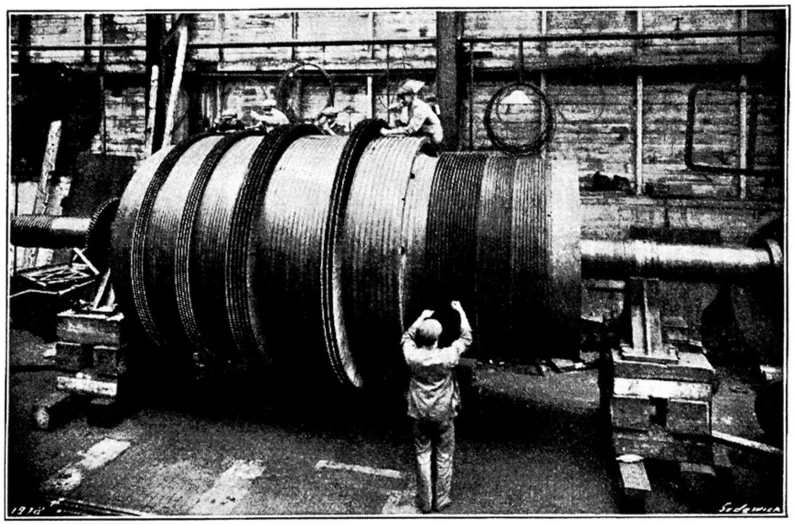
(b) This amount of heat is consistent with the fact that you warm quickly when exercising. Since the body is inefficient, the excess heat produced must be dissipated through sweating, breathing, etc.

Glossary

first law of thermodynamics	states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system
internal energy	the sum of the kinetic and potential energies of a system’s atoms and molecules
human metabolism	conversion of food into heat transfer, work, and stored fat

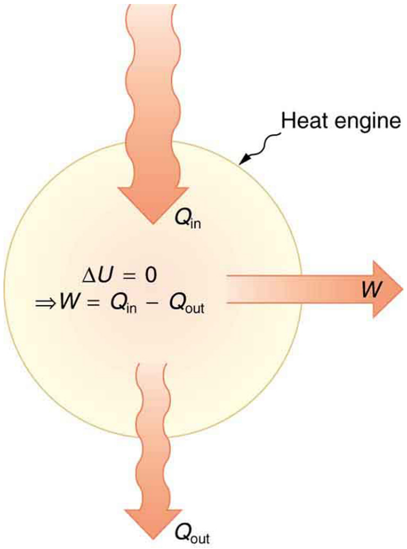
The First Law of Thermodynamics and Some Simple Processes

- Describe the processes of a simple heat engine.
- Explain the differences among the simple thermodynamic processes—isobaric, isochoric, isothermal, and adiabatic.
- Calculate total work done in a cyclical thermodynamic process.

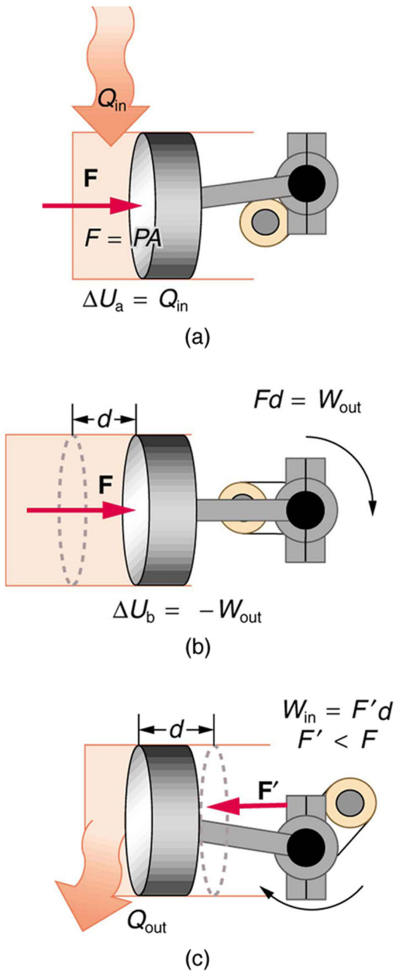


Beginning with the Industrial Revolution, humans have harnessed power through the use of the first law of thermodynamics, before we even understood it completely. This photo, of a steam engine at the Turbinia Works, dates from 1911, a mere 61 years after the first explicit statement of the first law of thermodynamics by Rudolph Clausius. (credit: public domain; author unknown)

One of the most important things we can do with heat transfer is to use it to do work for us. Such a device is called a **heat engine**. Car engines and steam turbines that generate electricity are examples of heat engines. [link] shows schematically how the first law of thermodynamics applies to the typical heat engine.


Heat engine
Q_{in}
$\Delta U = 0$
$\Rightarrow W = Q_{in} - Q_{out}$
W
Q_{out}

Schematic representation of a heat engine, governed, of course, by the first law of thermodynamics. It is impossible to devise a system where $Q_{\text{out}} = 0$, that is, in which no heat transfer occurs to the environment.


Q_{in}
F
$F = PA$
$\Delta U_a = Q_{in}$
(a)
$Fd = W_{out}$
d
F
$\Delta U_b = -W_{out}$
(b)
$W_{in} = F'd$
$F' < F$
d
F′
Q_{out}
(c)

(a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease when it expands, indicating that the gas’s internal energy has been decreased by doing work. (c) Heat transfer to the environment further reduces pressure in the gas so that the piston can be more easily returned to its starting position.

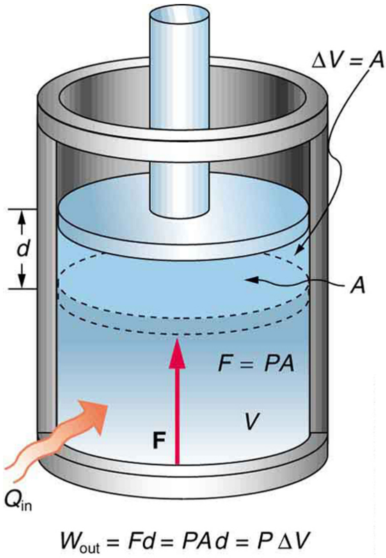
The illustrations above show one of the ways in which heat transfer does work. Fuel combustion produces heat transfer to a gas in a cylinder, increasing the pressure of the gas and thereby the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Heat transfer to the gas cylinder results in work being done. To repeat this process, the piston needs to be returned to its starting point. Heat transfer now occurs from the gas to the surroundings so that its pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance. Variations of this process are employed daily in hundreds of millions of heat engines. We will examine heat engines in detail in the next section. In this section, we consider some of the simpler underlying processes on which heat engines are based.

PV Diagrams and their Relationship to Work Done on or by a Gas

A process by which a gas does work on a piston at constant pressure is called an **isobaric process**. Since the pressure is constant, the force exerted is constant and the work done is given as

Equation:

$$P\Delta V.$$



$\Delta V = Ad$
d
A
$F = PA$
V
F
Q_{in}
$W_{out} = Fd = PAd = P\Delta V$

An isobaric expansion of a gas requires heat transfer to keep the pressure constant. Since pressure is constant, the work done is $P\Delta V$.

Equation:

$$W = \text{Fd}$$

See the symbols as shown in [link]. Now $F = \text{PA}$, and so

Equation:

$$W = \text{PAd}.$$

Because the volume of a cylinder is its cross-sectional area A times its length d, we see that $\mathrm{Ad} = \Delta V$, the change in volume; thus,

Equation:

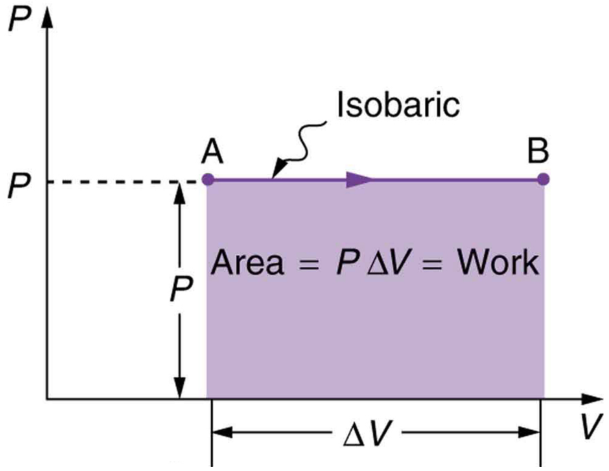
$$W = P\Delta V \text{ (isobaric process)}.$$

Note that if ΔV is positive, then W is positive, meaning that work is done *by* the gas on the outside world.

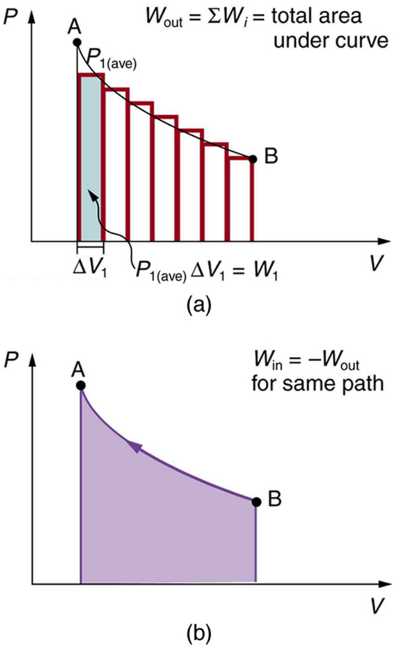
(Note that the pressure involved in this work that we’ve called P is the pressure of the gas *inside* the tank. If we call the pressure outside the tank P_{ext}, an expanding gas would be working *against* the external pressure; the work done would therefore be $W = -P_{\text{ext}}\Delta V$ (isobaric process). Many texts use this definition of work, and not the definition based on internal pressure, as the basis of the First Law of Thermodynamics. This definition reverses the sign conventions for work, and results in a statement of the first law that becomes $\Delta U = Q + W$.)

It is not surprising that $W = P\Delta V$, since we have already noted in our treatment of fluids that pressure is a type of potential energy per unit volume and that pressure in fact has units of energy divided by volume. We also noted in our discussion of the ideal gas law that PV has units of energy. In this case, some of the energy associated with pressure becomes work.

[link] shows a graph of pressure versus volume (that is, a PV diagram for an isobaric process. You can see in the figure that the work done is the area under the graph. This property of PV diagrams is very useful and broadly applicable: *the work done on or by a system in going from one state to another equals the area under the curve on a* PV *diagram*.


P
Isobaric
A
B
P
Area = $P\Delta V$ = Work
P
ΔV
V

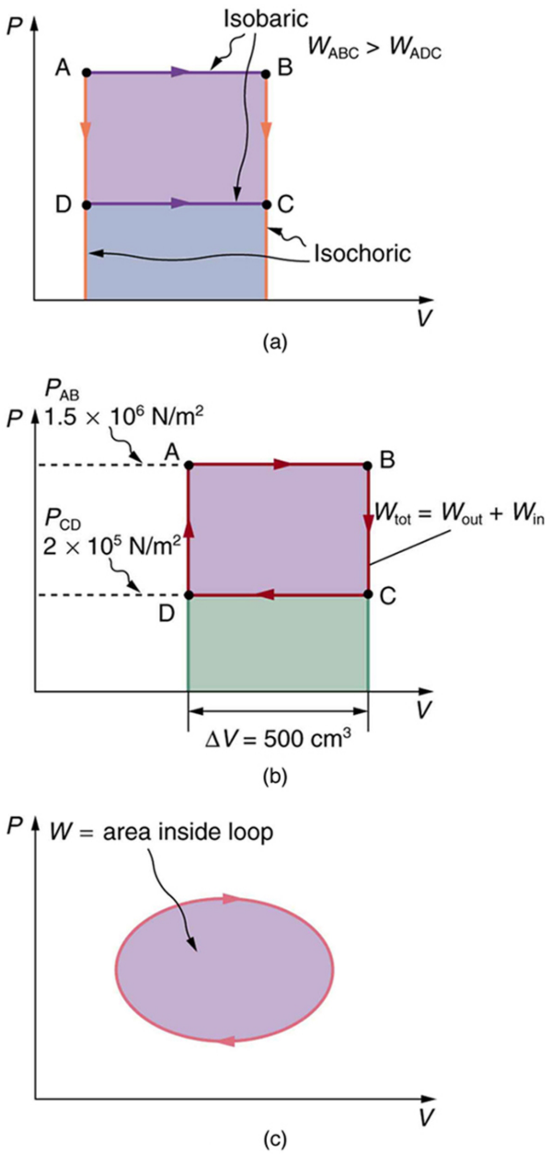
A graph of pressure versus volume for a constant-pressure, or isobaric, process, such as the one shown in [link]. The area under the curve equals the work done by the gas, since $W = P\Delta V.$


P
A
$W_{out} = \Sigma W_i$ = total area under curve
$P_{1(ave)}$
B
ΔV_1
$P_{1(ave)} \Delta V_1 = W_1$
V
(a)
P
A
$W_{in} = -W_{out}$ for same path
B
V
(b)

(a) A PV diagram in which pressure varies as well as volume. The work done for each interval is its average pressure times the change in volume, or the area under the curve over that interval. Thus the total area under the curve equals the total work done. (b) Work must be done on the system to follow the reverse path. This is interpreted as a negative area under the curve.

We can see where this leads by considering [link](a), which shows a more general process in which both pressure and volume change. The area under the curve is closely approximated by dividing it into strips, each having an average constant pressure $P_{i(\text{ave})}$. The work done is $W_i = P_{i(\text{ave})}\Delta V_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve. If the path is reversed, as in [link](b), then work is done on the system. The area under the curve in that case is negative, because ΔV is negative.

PV diagrams clearly illustrate that *the work done depends on the path taken and not just the endpoints*. This path dependence is seen in [link](a), where more work is done in going from A to C by the path via point B than by the path via point D. The vertical paths, where volume is constant, are called **isochoric** processes. Since volume is constant, $\Delta V = 0$, and no work is done in an isochoric process. Now, if the system follows the cyclical path ABCDA, as in [link](b), then the total work done is the area inside the loop. The negative area below path CD subtracts, leaving only the area inside the rectangle. In fact, the work done in any cyclical process (one that returns to its starting point) is the area inside the loop it forms on a PV diagram, as [link](c) illustrates for a general cyclical process. Note that the loop must be traversed in the clockwise direction for work to be positive—that is, for there to be a net work output.


P
Isobaric
$W_{ABC} > W_{ADC}$
A
B
D
C
Isochoric
V
(a)
P_{AB}
P
1.5×10^6 N/m^2
A
B
P_{CD}
2×10^5 N/m^2
$W_{tot} = W_{out} + W_{in}$
D
C
V
$\Delta V = 500$ cm^3
(b)
P
W = area inside loop
V
(c)

(a) The work done in going from A to C depends on path. The work is greater for the path ABC than for the path ADC, because the former is at higher pressure. In both cases, the work done is the area under the path. This area is greater for path ABC. (b) The total work done in the cyclical process

ABCDA is the area inside the loop, since the negative area below CD subtracts out, leaving just the area inside the rectangle. (The values given for the pressures and the change in volume are intended for use in the example below.) (c) The area inside any closed loop is the work done in the cyclical process. If the loop is traversed in a clockwise direction, W is positive—it is work done on the outside environment. If the loop is traveled in a counter-clockwise direction, W is negative—it is work that is done to the system.

Example:

Total Work Done in a Cyclical Process Equals the Area Inside the Closed Loop on a *PV* Diagram

Calculate the total work done in the cyclical process ABCDA shown in [link](b) by the following two methods to verify that work equals the area inside the closed loop on the PV diagram. (Take the data in the figure to be precise to three significant figures.) (a) Calculate the work done along each segment of the path and add these values to get the total work. (b) Calculate the area inside the rectangle ABCDA.

Strategy

To find the work along any path on a PV diagram, you use the fact that work is pressure times change in volume, or $W = P\Delta V$. So in part (a), this value is calculated for each leg of the path around the closed loop.

Solution for (a)

The work along path AB is

Equation:

$$\begin{aligned} W_{\text{AB}} &= P_{\text{AB}}\Delta V_{\text{AB}} \\ &= (1.50\times 10^{6}\ \text{N/m}^{2})(5.00\times 10^{-4}\ \text{m}^{3}) = 750\ \text{J}. \end{aligned}$$

Since the path BC is isochoric, $\Delta V_{\text{BC}} = 0$, and so $W_{\text{BC}} = 0$. The work along path CD is negative, since ΔV_{CD} is negative (the volume decreases). The work is

Equation:

$$\begin{aligned} W_{\text{CD}} &= P_{\text{CD}}\Delta V_{\text{CD}} \\ &= (2.00\times 10^{5}\ \text{N/m}^{2})(-5.00\times 10^{-4}\ \text{m}^{3}) = -100\ \text{J}. \end{aligned}$$

Again, since the path DA is isochoric, $\Delta V_{\text{DA}} = 0$, and so $W_{\text{DA}} = 0$. Now the total work is

Equation:

$$\begin{aligned} W &= W_{\text{AB}} + W_{\text{BC}} + W_{\text{CD}} + W_{\text{DA}} \\ &= 750\ \text{J} + 0 + (-100\text{J}) + 0 = 650\ \text{J}. \end{aligned}$$

Solution for (b)

The area inside the rectangle is its height times its width, or

Equation:

$$\begin{aligned} \text{area} &= (P_{\text{AB}} - P_{\text{CD}})\Delta V \\ &= \left[(1.50\times 10^{6}\ \text{N/m}^{2}) - (2.00\times 10^{5}\ \text{N/m}^{2})\right](5.00\times 10^{-4}\ \text{m}^{3}) \\ &= 650\ \text{J}. \end{aligned}$$

Thus,

Equation:

$$\text{area} = 650\ \text{J} = W.$$

Discussion

The result, as anticipated, is that the area inside the closed loop equals the work done. The area is often easier to calculate than is the work done along each path. It is also convenient to visualize the area inside different curves on PV diagrams in order to see which processes might produce the most work. Recall that work can be done to the system, or by the system, depending on the sign of W. A positive W is work that is done by the system on the outside environment; a negative W represents work done by the environment on the system.

[link](a) shows two other important processes on a PV diagram. For comparison, both are shown starting from the same point A. The upper curve ending at point B is an **isothermal** process—that is, one in which temperature is kept constant. If the gas behaves like an ideal gas, as is often the case, and if no phase change occurs, then $\text{PV} = \text{nRT}$. Since T is constant, PV is a constant for an isothermal process. We ordinarily expect the temperature of a gas to decrease as it expands, and so we correctly suspect that heat transfer must occur from the surroundings to the gas to keep the temperature constant during an isothermal expansion. To show this more rigorously for the special case of a monatomic ideal gas, we note that the average kinetic energy of an atom in such a gas is given by

Equation:

$$\frac{1}{2}mv^2 = \frac{3}{2}\text{kT}.$$

The kinetic energy of the atoms in a monatomic ideal gas is its only form of internal energy, and so its total internal energy U is

Equation:

$$U = N\frac{1}{2}mv^2 = \frac{3}{2}\text{NkT}, \text{(monatomic ideal gas)},$$

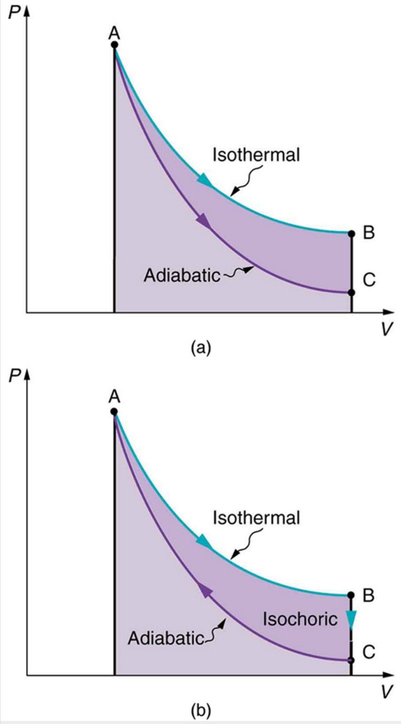
where N is the number of atoms in the gas. This relationship means that the internal energy of an ideal monatomic gas is constant during an isothermal process—that is, $\Delta U = 0$. If the internal energy does not change, then the net heat transfer into the gas must equal the net work done by the gas. That is, because $\Delta U = Q - W = 0$ here, $Q = W$. We must have just enough heat transfer to replace the work done. An isothermal process is inherently slow, because heat transfer occurs continuously to keep the gas temperature constant at all times and must be allowed to spread through the gas so that there are no hot or cold regions.

Also shown in [link](a) is a curve AC for an **adiabatic** process, defined to be one in which there is no heat transfer—that is, $Q=0$. Processes that are nearly adiabatic can be achieved either by using very effective insulation or by performing the process so fast that there is little time for heat transfer. Temperature must decrease during an adiabatic expansion process, since work is done at the expense of internal energy:

Equation:

$$U=\frac{3}{2}\mathrm{NkT}.$$

(You might have noted that a gas released into atmospheric pressure from a pressurized cylinder is substantially colder than the gas in the cylinder.) In fact, because $Q=0$, $\Delta U=-W$ for an adiabatic process. Lower temperature results in lower pressure along the way, so that curve AC is lower than curve AB, and less work is done. If the path ABCA could be followed by cooling the gas from B to C at constant volume (isochorically), [link](b), there would be a net work output.



(a) The upper curve is an isothermal process ($\Delta T = 0$), whereas the lower curve is an adiabatic process ($Q = 0$). Both start from the same point A, but the isothermal process does more work than the adiabatic because heat transfer into the gas takes place to keep its temperature constant. This keeps the pressure higher all along the isothermal path than along the adiabatic path, producing more work. The adiabatic path thus ends up with a lower pressure and temperature at point C, even though the final volume is the same as for the isothermal process. (b) The cycle ABCA produces a net work output.

Reversible Processes

Both isothermal and adiabatic processes such as shown in [link] are reversible in principle. A **reversible process** is one in which both the system and its environment can return to exactly the states they were in by following the reverse path. The reverse isothermal and adiabatic paths are BA and CA, respectively. Real macroscopic processes are never exactly reversible. In the previous examples, our system is a gas (like that in [link]), and its environment is the piston, cylinder, and the rest of the universe. If there are any energy-dissipating mechanisms, such as friction or turbulence, then heat transfer to the environment occurs for either direction of the piston. So, for example, if the path BA is followed and there is friction, then the gas will be returned to its original state but the environment will not—it will have been heated in both directions. Reversibility requires the direction of heat transfer to reverse for the reverse path. Since dissipative mechanisms cannot be completely eliminated, real processes cannot be reversible.

There must be reasons that real macroscopic processes cannot be reversible. We can imagine them going in reverse. For example, heat transfer occurs spontaneously from hot to cold and never spontaneously the reverse. Yet it would not violate the first law of thermodynamics for this to happen. In fact, all spontaneous processes, such as bubbles bursting, never go in reverse. There is a second thermodynamic law that forbids them from going in reverse. When we study this law, we will learn something about nature and also find that such a law limits the efficiency of heat engines. We will find that heat engines with the greatest possible theoretical efficiency would have to use reversible processes, and even they cannot convert all heat transfer into doing work. [link] summarizes the simpler thermodynamic processes and their definitions.

Isobaric	Constant pressure $W = P\Delta V$
Isochoric	Constant volume $W = 0$
Isothermal	Constant temperature $Q = W$
Adiabatic	No heat transfer $Q = 0$

Summary of Simple Thermodynamic Processes

Note:
PhET Explorations: States of Matter
Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.
https://phet.colorado.edu/sims/html/states-of-matter/latest/states-of-matter_en.html

Section Summary

- One of the important implications of the first law of thermodynamics is that machines can be harnessed to do work that humans previously did by hand or by external energy supplies such as running water or the heat of the Sun. A machine that uses heat transfer to do work is known as a heat engine.

- There are several simple processes, used by heat engines, that flow from the first law of thermodynamics. Among them are the isobaric, isochoric, isothermal and adiabatic processes.
- These processes differ from one another based on how they affect pressure, volume, temperature, and heat transfer.
- If the work done is performed on the outside environment, work (W) will be a positive value. If the work done is done to the heat engine system, work (W) will be a negative value.
- Some thermodynamic processes, including isothermal and adiabatic processes, are reversible in theory; that is, both the thermodynamic system and the environment can be returned to their initial states. However, because of loss of energy owing to the second law of thermodynamics, complete reversibility does not work in practice.

Conceptual Questions

Exercise:

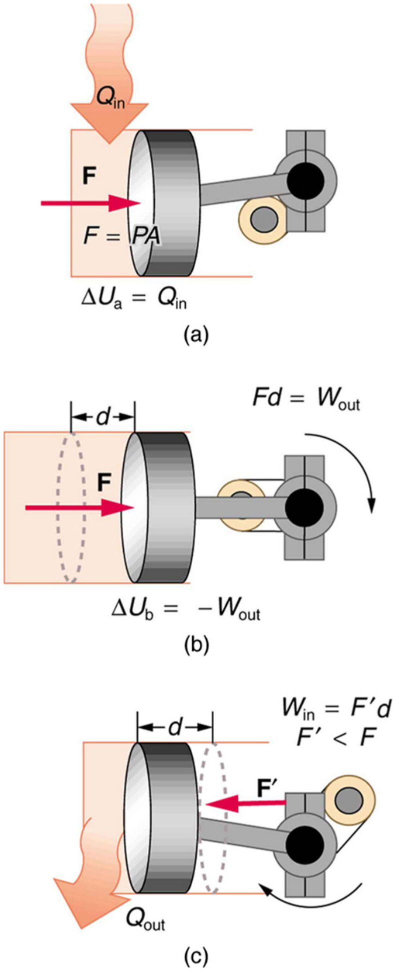
Problem:

A great deal of effort, time, and money has been spent in the quest for the so-called perpetual-motion machine, which is defined as a hypothetical machine that operates or produces useful work indefinitely and/or a hypothetical machine that produces more work or energy than it consumes. Explain, in terms of heat engines and the first law of thermodynamics, why or why not such a machine is likely to be constructed.

Exercise:

Problem:

One method of converting heat transfer into doing work is for heat transfer into a gas to take place, which expands, doing work on a piston, as shown in the figure below. (a) Is the heat transfer converted directly to work in an isobaric process, or does it go through another form first? Explain your answer. (b) What about in an isothermal process? (c) What about in an adiabatic process (where heat transfer occurred prior to the adiabatic process)?


Q_{in}
F
$F = PA$
$\Delta U_a = Q_{in}$
(a)
d
$Fd = W_{out}$
F
$\Delta U_b = -W_{out}$
(b)
d
$W_{in} = F'd$
$F' < F$
F′
Q_{out}
(c)

Exercise:

Problem:

Would the previous question make any sense for an isochoric process? Explain your answer.

Exercise:

Problem:

We ordinarily say that $\Delta U = 0$ for an isothermal process. Does this assume no phase change takes place? Explain your answer.

Exercise:

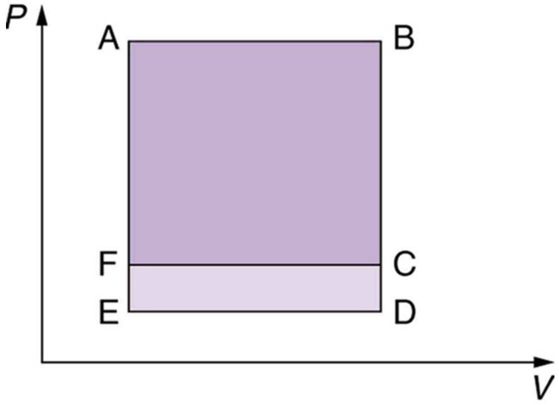
Problem:

The temperature of a rapidly expanding gas decreases. Explain why in terms of the first law of thermodynamics. (Hint: Consider whether the gas does work and whether heat transfer occurs rapidly into the gas through conduction.)

Exercise:

Problem:

Which cyclical process represented by the two closed loops, ABCFA and ABDEA, on the PV diagram in the figure below produces the greatest *net* work? Is that process also the one with the smallest work input required to return it to point A? Explain your responses.



The two cyclical processes shown on this PV diagram start with and return the system to the conditions at point A, but they follow

different paths and produce
different amounts of work.

Exercise:

Problem:

A real process may be nearly adiabatic if it occurs over a very short time. How does the short time span help the process to be adiabatic?

Exercise:

Problem:

It is unlikely that a process can be isothermal unless it is a very slow process. Explain why. Is the same true for isobaric and isochoric processes? Explain your answer.

Problem Exercises

Exercise:

Problem:

A car tire contains 0.0380 m^3 of air at a pressure of $2.20 \times 10^5 \text{ N/m}^2$ (about 32 psi). How much more internal energy does this gas have than the same volume has at zero gauge pressure (which is equivalent to normal atmospheric pressure)?

Solution:

$6.77 \times 10^3 \text{ J}$

Exercise:

Problem:

A helium-filled toy balloon has a gauge pressure of 0.200 atm and a volume of 10.0 L. How much greater is the internal energy of the helium in the balloon than it would be at zero gauge pressure?

Exercise:

Problem:

Steam to drive an old-fashioned steam locomotive is supplied at a constant gauge pressure of $1.75\times10^{6}\ \text{N/m}^{2}$ (about 250 psi) to a piston with a 0.200-m radius. (a) By calculating $P\Delta V$, find the work done by the steam when the piston moves 0.800 m. Note that this is the net work output, since gauge pressure is used. (b) Now find the amount of work by calculating the force exerted times the distance traveled. Is the answer the same as in part (a)?

Solution:

(a) $W = P\Delta V = 1.76 \times 10^{5}\ \text{J}$

(b) $W = \text{Fd} = 1.76 \times 10^{5}\ \text{J}$. Yes, the answer is the same.

Exercise:

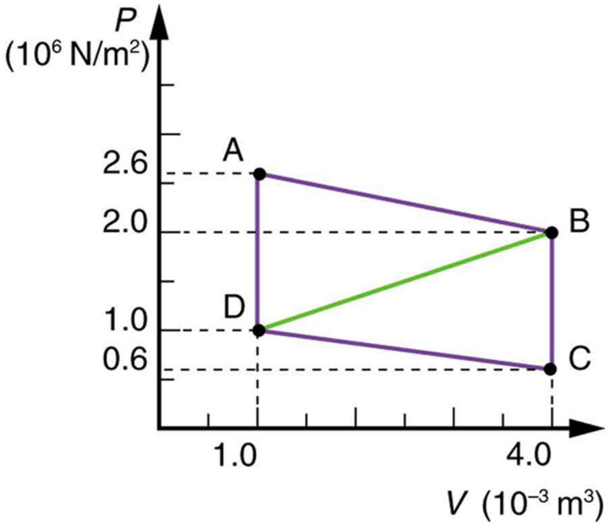
Problem:

A hand-driven tire pump has a piston with a 2.50-cm diameter and a maximum stroke of 30.0 cm. (a) How much work do you do in one stroke if the average gauge pressure is $2.40\times10^{5}\ \text{N/m}^{2}$ (about 35 psi)? (b) What average force do you exert on the piston, neglecting friction and gravitational force?

Exercise:

Problem:

Calculate the net work output of a heat engine following path ABCDA in the figure below.



Solution:

$W = 4.5 \times 10^3 \text{ J}$

Exercise:

Problem:

What is the net work output of a heat engine that follows path ABDA in the figure above, with a straight line from B to D? Why is the work output less than for path ABCDA? Explicitly show how you follow the steps in the Problem-Solving Strategies for Thermodynamics.

Exercise:

Problem: Unreasonable Results

What is wrong with the claim that a cyclical heat engine does 4.00 kJ of work on an input of 24.0 kJ of heat transfer while 16.0 kJ of heat transfers to the environment?

Solution:

W is not equal to the difference between the heat input and the heat output.

Exercise:

Problem:

(a) A cyclical heat engine, operating between temperatures of 450° C and 150° C produces 4.00 MJ of work on a heat transfer of 5.00 MJ into the engine. How much heat transfer occurs to the environment? (b) What is unreasonable about the engine? (c) Which premise is unreasonable?

Exercise:

Problem: Construct Your Own Problem

Consider a car's gasoline engine. Construct a problem in which you calculate the maximum efficiency this engine can have. Among the things to consider are the effective hot and cold reservoir temperatures. Compare your calculated efficiency with the actual efficiency of car engines.

Exercise:

Problem: Construct Your Own Problem

Consider a car trip into the mountains. Construct a problem in which you calculate the overall efficiency of the car for the trip as a ratio of kinetic and potential energy gained to fuel consumed. Compare this efficiency to the thermodynamic efficiency quoted for gasoline engines and discuss why the thermodynamic efficiency is so much greater. Among the factors to be considered are the gain in altitude and speed, the mass of the car, the distance traveled, and typical fuel economy.

Glossary

heat engine
    a machine that uses heat transfer to do work

isobaric process
    constant-pressure process in which a gas does work

isochoric process
a constant-volume process

isothermal process
a constant-temperature process

adiabatic process
a process in which no heat transfer takes place

reversible process
a process in which both the heat engine system and the external environment theoretically can be returned to their original states

The Ideal Gas Law

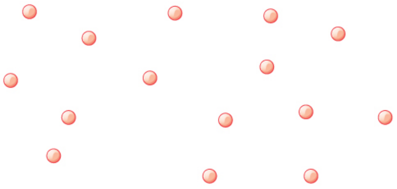
- State the ideal gas law in terms of molecules and in terms of moles.
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules or moles in a given volume.
- Use Avogadro’s number to convert between number of molecules and number of moles.

The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2, and oxygen, O_2, are composed of two or more atoms. We will primarily use the term “molecule” in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

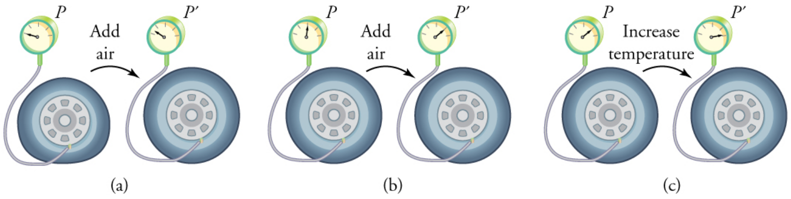
Gases are easily compressed. We can see evidence of this in [link], where you will note that gases have the *largest* coefficients of volume expansion. The large coefficients mean that gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β. This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in [link]. Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.



Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See [link].)



(a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

Note:
Ideal Gas Law
The **ideal gas law** states that
Equation:

$$\mathrm{PV} = \mathrm{NkT},$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value
Equation:

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V. The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $\mathrm{PV} = \mathrm{NkT}$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example:
Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of $7.00 \times 10^5\ \text{Pa}$ (a gauge pressure of just under $90.0\ \text{lb/in}^2$) at a temperature of 18.0^{o}C. What is the pressure after its temperature has risen to 35.0^{o}C? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5\ \text{Pa}$, the initial temperature $T_0 = 18.0^{\text{o}}\text{C}$, and the final temperature $T_{\text{f}} = 35.0^{\text{o}}\text{C}$. We must find the final pressure P_{f}. How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0 V_0 = NkT_0$ and $P_{\text{f}} V_{\text{f}} = NkT_{\text{f}}$. If we divide $P_{\text{f}} V_{\text{f}}$ by $P_0 V_0$ we can come up with an equation that allows us to solve for P_{f}.

Equation:

$$\frac{P_{\text{f}} V_{\text{f}}}{P_0 V_0} = \frac{N_{\text{f}} k T_{\text{f}}}{N_0 k T_0}$$

Since the volume is constant, V_{f} and V_0 are the same and they cancel out. The same is true for N_{f} and N_0, and k, which is a constant. Therefore,

Equation:

$$\frac{P_{\text{f}}}{P_0} = \frac{T_{\text{f}}}{T_0}.$$

We can then rearrange this to solve for P_{f}:

Equation:

$$P_{\text{f}} = P_0 \frac{T_{\text{f}}}{T_0},$$

where the temperature must be in units of kelvins, because T_0 and T_{f} are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.

Equation:

$$\begin{aligned} T_0 &= (18.0 + 273)\text{K} = 291\ \text{K} \\ T_{\text{f}} &= (35.0 + 273)\text{K} = 308\ \text{K} \end{aligned}$$

2. Substitute the known values into the equation.

Equation:

$$P_{\text{f}} = P_0 \frac{T_{\text{f}}}{T_0} = 7.00 \times 10^5\ \text{Pa} \left(\frac{308\ \text{K}}{291\ \text{K}} \right) = 7.41 \times 10^5\ \text{Pa}$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Note:

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example:

Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be 0ºC and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $\text{PV} = \text{NkT}$, to find N.

Solution

1. Identify the knowns.

Equation:

$$\begin{aligned} T &= 0^\circ\text{C} = 273\ \text{K} \\ P &= 1.01 \times 10^5\ \text{Pa} \\ V &= 1.00\ \text{m}^3 \\ k &= 1.38 \times 10^{-23}\ \text{J/K} \end{aligned}$$

2. Identify the unknown: number of molecules, N.

3. Rearrange the ideal gas law to solve for N.

Equation:

$$\text{PV} = \text{NkT}$$
$$N = \frac{\text{PV}}{\text{kT}}$$

4. Substitute the known values into the equation and solve for N.

Equation:

$$N = \frac{\text{PV}}{\text{kT}} = \frac{\left(1.01 \times 10^5\ \text{Pa}\right)\left(1.00\ \text{m}^3\right)}{\left(1.38 \times 10^{-23}\ \text{J/K}\right)(273\ \text{K})} = 2.68 \times 10^{25}\ \text{molecules}$$

Discussion

This number is undeniably large, considering that a gas is mostly empty space. N is huge, even in small volumes. For example, 1 cm^3 of a gas at STP has 2.68×10^{19} molecules in it. Once again, note that N is the same for all types or mixtures of gases.

Moles and Avogadro's Number

It is sometimes convenient to work with a unit other than molecules when measuring the amount of substance. A **mole** (abbreviated mol) is defined to be the amount of a substance that contains as many atoms or molecules as there are atoms in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called **Avogadro's number**(N_{A}), in recognition of Italian scientist Amedeo Avogadro (1776–1856). He developed the concept of the mole, based on the hypothesis that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules. That is, the number is independent of the type of gas. This hypothesis has been confirmed, and the value of Avogadro's number is

Equation:

$$N_{\text{A}} = 6.02 \times 10^{23}\ \text{mol}^{-1}.$$

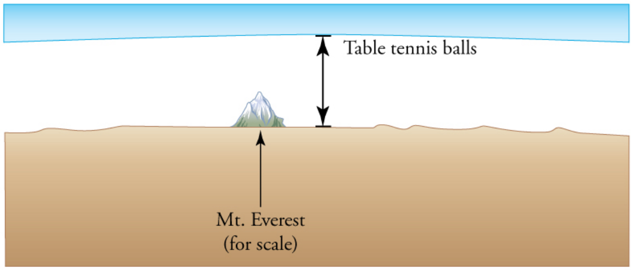
Note:

Avogadro's Number

One mole always contains 6.02×10^{23} particles (atoms or molecules), independent of the element or substance. A mole of any substance has a mass in grams equal to its molecular mass, which can be calculated from the atomic masses given in the periodic table of elements.

Equation:

$$N_{\text{A}} = 6.02 \times 10^{23}\ \text{mol}^{-1}$$



How big is a mole? On a macroscopic level, one mole of table tennis balls would cover the Earth to a depth of about 40 km.

Exercise:
Check Your Understanding

Problem:

The active ingredient in a Tylenol pill is 325 mg of acetaminophen $(C_8H_9NO_2)$. Find the number of active molecules of acetaminophen in a single pill.

Solution:

We first need to calculate the molar mass (the mass of one mole) of acetaminophen. To do this, we need to multiply the number of atoms of each element by the element’s atomic mass.
Equation:

$$\begin{array}{l}(8\text{ moles of carbon})(12\text{ grams/mole})+(9\text{ moles hydrogen})(1\text{ gram/mole})\\ +(1\text{ mole nitrogen})(14\text{ grams/mole})+(2\text{ moles oxygen})(16\text{ grams/mole})=151\text{ g}\end{array}$$

Then we need to calculate the number of moles in 325 mg.
Equation:

$$\left(\frac{325\text{ mg}}{151\text{ grams/mole}}\right)\left(\frac{1\text{ gram}}{1000\text{ mg}}\right)=2.15\times10^{-3}\text{ moles}$$

Then use Avogadro’s number to calculate the number of molecules.
Equation:

$$N=\left(2.15\times10^{-3}\text{ moles}\right)\left(6.02\times10^{23}\text{ molecules/mole}\right)=1.30\times10^{21}\text{ molecules}$$

Example:
Calculating Moles per Cubic Meter and Liters per Mole
Calculate: (a) the number of moles in 1.00 m^3 of gas at STP, and (b) the number of liters of gas per mole.
Strategy and Solution
(a) We are asked to find the number of moles per cubic meter, and we know from [link] that the number of molecules per cubic meter at STP is 2.68×10^{25}. The number of moles can be found by dividing the number of molecules by Avogadro’s number. We let n stand for the number of moles,
Equation:

$$n\text{ mol/m}^3=\frac{N\text{ molecules/m}^3}{6.02\times10^{23}\text{ molecules/mol}}=\frac{2.68\times10^{25}\text{ molecules/m}^3}{6.02\times10^{23}\text{ molecules/mol}}=44.5\text{ mol/m}^3.$$

(b) Using the value obtained for the number of moles in a cubic meter, and converting cubic meters to liters, we obtain

Equation:

$$\frac{\left(10^3\ \text{L/m}^3\right)}{44.5\ \text{mol/m}^3} = 22.5\ \text{L/mol.}$$

Discussion

This value is very close to the accepted value of 22.4 L/mol. The slight difference is due to rounding errors caused by using three-digit input. Again this number is the same for all gases. In other words, it is independent of the gas.

The (average) molar weight of air (approximately 80% N_2 and 20% O_2 is $M = 28.8\ \text{g}$. Thus the mass of one cubic meter of air is 1.28 kg. If a living room has dimensions $5\ \text{m} \times 5\ \text{m} \times 3\ \text{m}$, the mass of air inside the room is 96 kg, which is the typical mass of a human.

Exercise:

Check Your Understanding

Problem:

The density of air at standard conditions $(P = 1\ \text{atm}$ and $T = 20^{\circ}\text{C})$ is $1.28\ \text{kg/m}^3$. At what pressure is the density $0.64\ \text{kg/m}^3$ if the temperature and number of molecules are kept constant?

Solution:

The best way to approach this question is to think about what is happening. If the density drops to half its original value and no molecules are lost, then the volume must double. If we look at the equation $\text{PV} = \text{NkT}$, we see that when the temperature is constant, the pressure is inversely proportional to volume. Therefore, if the volume doubles, the pressure must drop to half its original value, and $P_{\text{f}} = 0.50\ \text{atm}$.

The Ideal Gas Law Restated Using Moles

A very common expression of the ideal gas law uses the number of moles, n, rather than the number of atoms and molecules, N. We start from the ideal gas law,

Equation:

$$\text{PV} = \text{NkT},$$

and multiply and divide the equation by Avogadro's number N_{A}. This gives

Equation:

$$\mathrm{PV} = \frac{N}{N_{\mathrm{A}}} N_{\mathrm{A}} \mathrm{kT}.$$

Note that $n = N/N_{\mathrm{A}}$ is the number of moles. We define the universal gas constant $R = N_{\mathrm{A}}k$, and obtain the ideal gas law in terms of moles.

Note:

Ideal Gas Law (in terms of moles)

The ideal gas law (in terms of moles) is

Equation:

$$\mathrm{PV} = \mathrm{nRT}.$$

The numerical value of R in SI units is

Equation:

$$R = N_{\mathrm{A}}k = \left(6.02 \times 10^{23}\ \mathrm{mol}^{-1}\right)\left(1.38 \times 10^{-23}\ \mathrm{J/K}\right) = 8.31\ \mathrm{J/mol \cdot K}.$$

In other units,

Equation:

$$\begin{aligned} R &= 1.99\ \mathrm{cal/mol \cdot K} \\ R &= 0.0821\ \mathrm{L \cdot atm/mol \cdot K}. \end{aligned}$$

You can use whichever value of R is most convenient for a particular problem.

Example:

Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of $2.00 \times 10^{-3}\ \mathrm{m}^3(2.00\ \mathrm{L})$, a pressure of $7.00 \times 10^5\ \mathrm{Pa}$ (a gauge pressure of just under $90.0\ \mathrm{lb/in}^2$), and at a temperature of $18.0^{\circ}\mathrm{C}$?

Strategy

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law, $\mathrm{PV} = \mathrm{nRT}$, for the number of moles n.

Solution

1. Identify the knowns.

Equation:

$$\begin{aligned} P &= 7.00 \times 10^5\ \mathrm{Pa} \\ V &= 2.00 \times 10^{-3}\ \mathrm{m}^3 \\ T &= 18.0^{\circ}\mathrm{C} = 291\ \mathrm{K} \\ R &= 8.31\ \mathrm{J/mol \cdot K} \end{aligned}$$

2. Rearrange the equation to solve for n and substitute known values.
Equation:

$$\begin{aligned} n &= \frac{\text{PV}}{\text{RT}} = \frac{\left(7.00\times10^{5}\ \text{Pa}\right)\left(2.00\times10^{-3}\ \text{m}^{3}\right)}{(8.31\ \text{J/mol}\cdot\text{K})(291\ \text{K})} \\ &= 0.579\ \text{mol} \end{aligned}$$

Discussion
The most convenient choice for R in this case is $8.31\ \text{J/mol}\cdot\text{K}$, because our known quantities are in SI units. The pressure and temperature are obtained from the initial conditions in [link], but we would get the same answer if we used the final values.

The ideal gas law can be considered to be another manifestation of the law of conservation of energy (see Conservation of Energy). Work done on a gas results in an increase in its energy, increasing pressure and/or temperature, or decreasing volume. This increased energy can also be viewed as increased internal kinetic energy, given the gas's atoms and molecules.

The Ideal Gas Law and Energy

Let us now examine the role of energy in the behavior of gases. When you inflate a bike tire by hand, you do work by repeatedly exerting a force through a distance. This energy goes into increasing the pressure of air inside the tire and increasing the temperature of the pump and the air.

The ideal gas law is closely related to energy: the units on both sides are joules. The right-hand side of the ideal gas law in $\text{PV} = \text{NkT}$ is NkT. This term is roughly the amount of translational kinetic energy of N atoms or molecules at an absolute temperature T, as we shall see formally in Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature. The left-hand side of the ideal gas law is PV, which also has the units of joules. We know from our study of fluids that pressure is one type of potential energy per unit volume, so pressure multiplied by volume is energy. The important point is that there is energy in a gas related to both its pressure and its volume. The energy can be changed when the gas is doing work as it expands—something we explore in Heat and Heat Transfer Methods—similar to what occurs in gasoline or steam engines and turbines.

Note:
Problem-Solving Strategy: The Ideal Gas Law
Step 1 Examine the situation to determine that an ideal gas is involved. Most gases are nearly ideal.
Step 2 Make a list of what quantities are given, or can be inferred from the problem as stated (identify the known quantities). Convert known values into proper SI units (K for temperature, Pa for pressure, m^3 for volume, molecules for N, and moles for n).
Step 3 Identify exactly what needs to be determined in the problem (identify the unknown quantities). A written list is useful.

Step 4 Determine whether the number of molecules or the number of moles is known, in order to decide which form of the ideal gas law to use. The first form is $\mathrm{PV} = \mathrm{NkT}$ and involves N, the number of atoms or molecules. The second form is $\mathrm{PV} = \mathrm{nRT}$ and involves n, the number of moles.
Step 5 Solve the ideal gas law for the quantity to be determined (the unknown quantity). You may need to take a ratio of final states to initial states to eliminate the unknown quantities that are kept fixed.
Step 6 Substitute the known quantities, along with their units, into the appropriate equation, and obtain numerical solutions complete with units. Be certain to use absolute temperature and absolute pressure.
Step 7 Check the answer to see if it is reasonable: Does it make sense?

Exercise:
Check Your Understanding

Problem:

Liquids and solids have densities about 1000 times greater than gases. Explain how this implies that the distances between atoms and molecules in gases are about 10 times greater than the size of their atoms and molecules.

Solution:

Atoms and molecules are close together in solids and liquids. In gases they are separated by empty space. Thus gases have lower densities than liquids and solids. Density is mass per unit volume, and volume is related to the size of a body (such as a sphere) cubed. So if the distance between atoms and molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000.

Section Summary

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:
  Equation:

  $$\mathrm{PV} = \mathrm{NkT},$$

  where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant
  Equation:

  $$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

- A mole is the number of atoms in a 12-g sample of carbon-12.
- The number of molecules in a mole is called Avogadro’s number N_{A},

Equation:

$$N_{\mathrm{A}} = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}.$$

- A mole of any substance has a mass in grams equal to its molecular weight, which can be determined from the periodic table of elements.
- The ideal gas law can also be written and solved in terms of the number of moles of gas:
  Equation:

  $$\mathrm{PV} = \mathrm{nRT},$$

  where n is number of moles and R is the universal gas constant,
  Equation:

  $$R = 8.31\ \mathrm{J/mol} \cdot \mathrm{K}.$$

- The ideal gas law is generally valid at temperatures well above the boiling temperature.

Conceptual Questions

Exercise:

Problem:

Find out the human population of Earth. Is there a mole of people inhabiting Earth? If the average mass of a person is 60 kg, calculate the mass of a mole of people. How does the mass of a mole of people compare with the mass of Earth?

Exercise:

Problem:

Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

Exercise:

Problem:

A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

Problems & Exercises

Exercise:

Problem:

The gauge pressure in your car tires is $2.50 \times 10^5\ \mathrm{N/m}^2$ at a temperature of $35.0^{\mathrm{o}}\mathrm{C}$ when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to $-40.0^{\mathrm{o}}\mathrm{C}$?

Solution:

1.62 atm

Exercise:

Problem:

Convert an absolute pressure of $7.00 \times 10^5\ \mathrm{N/m^2}$ to gauge pressure in $\mathrm{lb/in^2}$. (This value was stated to be just less than $90.0\ \mathrm{lb/in^2}$ in [link]. Is it?)

Exercise:

Problem:

Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of $20.0^\circ\mathrm{C}$. (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is $60.0^\circ\mathrm{C}$ (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

Solution:

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

Exercise:

Problem:

Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of $10.0^\circ\mathrm{C}$ and rises to an altitude where its volume is twenty times the original volume and its temperature is $-50.0^\circ\mathrm{C}$? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

Exercise:

Problem:

Confirm that the units of nRT are those of energy for each value of R: (a) $8.31\ \mathrm{J/mol \cdot K}$, (b) $1.99\ \mathrm{cal/mol \cdot K}$, and (c) $0.0821\ \mathrm{L \cdot atm/mol \cdot K}$.

Solution:

(a) $\mathrm{nRT} = (\mathrm{mol})(\mathrm{J/mol \cdot K})(\mathrm{K}) = \mathrm{J}$

(b) $\mathrm{nRT} = (\mathrm{mol})(\mathrm{cal/mol \cdot K})(\mathrm{K}) = \mathrm{cal}$

$$\begin{array}{lrcl} & \mathrm{nRT} & = & (\mathrm{mol})(\mathrm{L}\cdot\mathrm{atm}/\mathrm{mol}\cdot\mathrm{K})(\mathrm{K}) \\ \text{(c)} & & = & \mathrm{L}\cdot\mathrm{atm} = (\mathrm{m}^3)(\mathrm{N}/\mathrm{m}^2) \\ & & = & \mathrm{N}\cdot\mathrm{m} = \mathrm{J} \end{array}$$

Exercise:

Problem:

In the text, it was shown that $N/V = 2.68 \times 10^{25}\ \mathrm{m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68 \times 10^{19}\ \mathrm{cm}^{-3}$, as stated. (b) About how many atoms are there in one $\mu\mathrm{m}^3$ (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

Exercise:

Problem:

Calculate the number of moles in the 2.00-L volume of air in the lungs of the average person. Note that the air is at $37.0^{\circ}\mathrm{C}$ (body temperature).

Solution:

$7.86 \times 10^{-2}\ \mathrm{mol}$

Exercise:

Problem:

An airplane passenger has $100\ \mathrm{cm}^3$ of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50 \times 10^4\ \mathrm{N/m}^2$?

Exercise:

Problem:

(a) What is the volume (in km^3) of Avogadro’s number of sand grains if each grain is a cube and has sides that are 1.0 mm long? (b) How many kilometers of beaches in length would this cover if the beach averages 100 m in width and 10.0 m in depth? Neglect air spaces between grains.

Solution:

(a) $6.02 \times 10^5\ \mathrm{km}^3$

(b) $6.02 \times 10^8\ \mathrm{km}$

Exercise:

Problem:

An expensive vacuum system can achieve a pressure as low as $1.00 \times 10^{-7}\ \mathrm{N/m^2}$ at $20^{\mathrm{o}}\mathrm{C}$. How many atoms are there in a cubic centimeter at this pressure and temperature?

Exercise:

Problem:

The number density of gas atoms at a certain location in the space above our planet is about $1.00 \times 10^{11}\ \mathrm{m}^{-3}$, and the pressure is $2.75 \times 10^{-10}\ \mathrm{N/m^2}$ in this space. What is the temperature there?

Solution:

$-73.9^{\mathrm{o}}\mathrm{C}$

Exercise:

Problem:

A bicycle tire has a pressure of $7.00 \times 10^{5}\ \mathrm{N/m^2}$ at a temperature of $18.0^{\mathrm{o}}\mathrm{C}$ and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of $100\ \mathrm{cm}^3$ at atmospheric pressure? Assume tire temperature and volume remain constant.

Exercise:

Problem:

A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40 \times 10^{7}\ \mathrm{N/m^2}$ and a temperature of $25.0^{\mathrm{o}}\mathrm{C}$. Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature $(–78.5^{\mathrm{o}}\mathrm{C})$ to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

Solution:

(a) $9.14 \times 10^{6}\ \mathrm{N/m^2}$

(b) $8.23 \times 10^{6}\ \mathrm{N/m^2}$

(c) 2.16 K

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

Exercise:

Problem:

Find the number of moles in 2.00 L of gas at $35.0^{\mathrm{o}}\mathrm{C}$ and under $7.41 \times 10^{7}\ \mathrm{N/m^{2}}$ of pressure.

Exercise:

Problem:

Calculate the depth to which Avogadro’s number of table tennis balls would cover Earth. Each ball has a diameter of 3.75 cm. Assume the space between balls adds an extra 25.0% to their volume and assume they are not crushed by their own weight.

Solution:

41 km

Exercise:

Problem:

(a) What is the gauge pressure in a $25.0^{\mathrm{o}}\mathrm{C}$ car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and $25.0^{\mathrm{o}}\mathrm{C}$? Assume the temperature returns to $25.0^{\mathrm{o}}\mathrm{C}$ and the volume remains constant.

Exercise:

Problem:

(a) In the deep space between galaxies, the density of atoms is as low as $10^{6}\ \mathrm{atoms/m^{3}}$, and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in $\mathrm{m^{3}}$) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?

Solution:

(a) $3.7 \times 10^{-17}\ \mathrm{Pa}$

(b) $6.0 \times 10^{17}\ \mathrm{m^{3}}$

(c) $8.4 \times 10^{2}\ \mathrm{km}$

Glossary

ideal gas law
: the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Boltzmann constant

k , a physical constant that relates energy to temperature; $k = 1.38 \times 10^{-23}\ \text{J/K}$

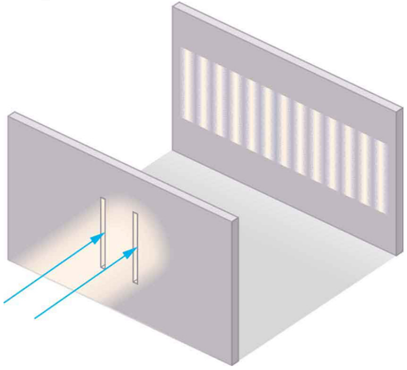
Avogadro’s number
: N_A , the number of molecules or atoms in one mole of a substance; $N_\text{A} = 6.02 \times 10^{23}$ particles/mole

mole
: the quantity of a substance whose mass (in grams) is equal to its molecular mass

Young’s Double Slit Experiment

- Explain the phenomena of interference.
- Define constructive interference for a double slit and destructive interference for a double slit.

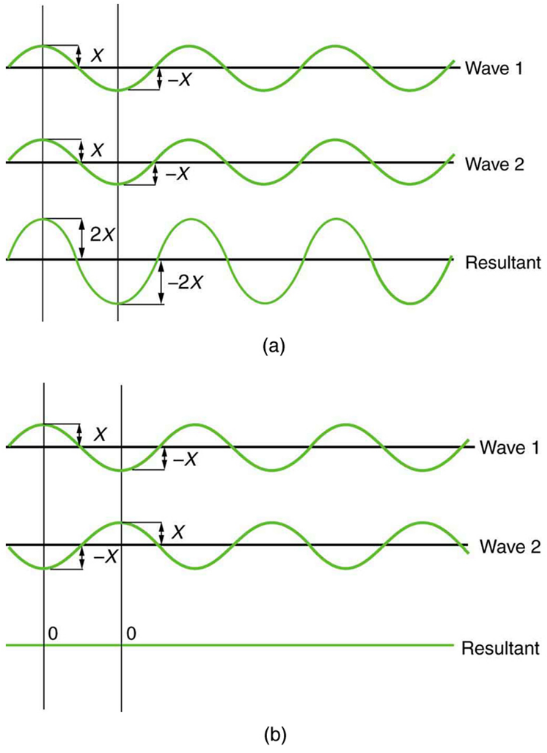
Although Christiaan Huygens thought that light was a wave, Isaac Newton did not. Newton felt that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton’s tremendous stature, his view generally prevailed. The fact that Huygens’s principle worked was not considered evidence that was direct enough to prove that light is a wave. The acceptance of the wave character of light came many years later when, in 1801, the English physicist and physician Thomas Young (1773–1829) did his now-classic double slit experiment (see [link]).



Young’s double slit experiment. Here pure-wavelength light sent through a pair of vertical slits is diffracted into a pattern on the screen of numerous vertical lines spread out horizontally. Without diffraction and interference, the light would

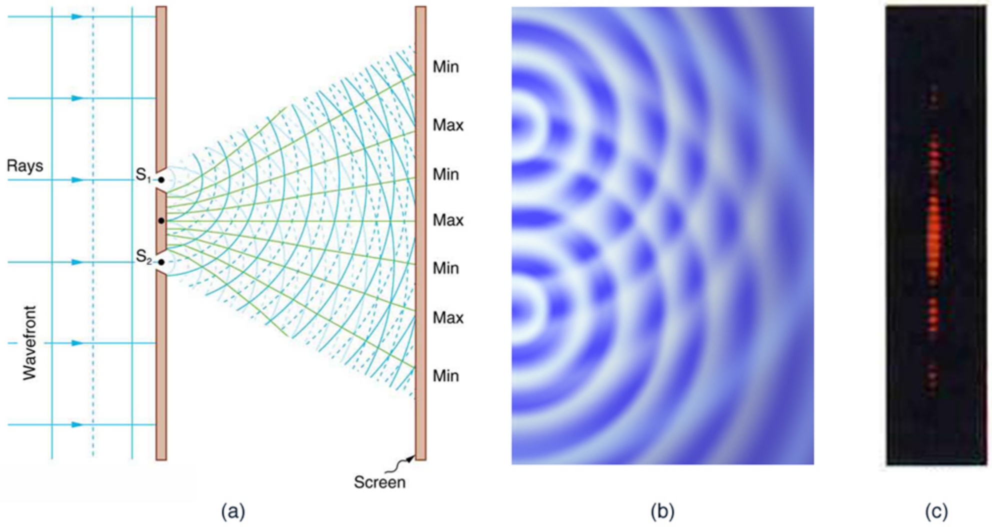
simply make two
lines on the screen.

Why do we not ordinarily observe wave behavior for light, such as observed in Young’s double slit experiment? First, light must interact with something small, such as the closely spaced slits used by Young, to show pronounced wave effects. Furthermore, Young first passed light from a single source (the Sun) through a single slit to make the light somewhat coherent. By **coherent**, we mean waves are in phase or have a definite phase relationship. **Incoherent** means the waves have random phase relationships. Why did Young then pass the light through a double slit? The answer to this question is that two slits provide two coherent light sources that then interfere constructively or destructively. Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. We illustrate the double slit experiment with monochromatic (single λ) light to clarify the effect. [link] shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.



The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in [link](a). Pure constructive interference occurs where the waves are crest to crest or trough to trough. Pure destructive interference occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in [link](b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.



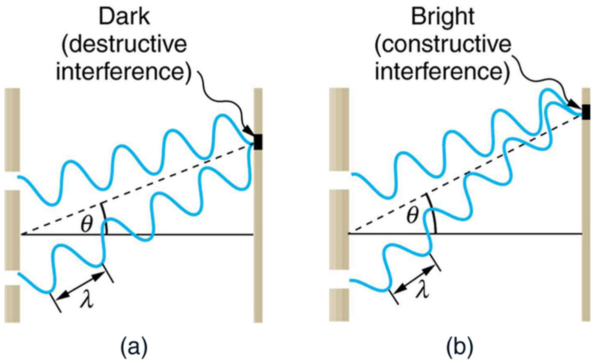
Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this. (credit: PASCO)

To understand the double slit interference pattern, we consider how two waves travel from the slits to the screen, as illustrated in [link]. Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively as shown in [link](a). If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively as shown in [link](b). More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths [$(1/2)\lambda$, $(3/2)\lambda$, $(5/2)\lambda$, etc.], then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths (λ, 2λ, 3λ, etc.), then constructive interference occurs.

Note:

Take-Home Experiment: Using Fingers as Slits

Look at a light, such as a street lamp or incandescent bulb, through the narrow gap between two fingers held close together. What type of pattern do you see? How does it change when you allow the fingers to move a little farther apart? Is it more distinct for a monochromatic source, such as the yellow light from a sodium vapor lamp, than for an incandescent bulb?



Waves follow different paths from the slits to a common point on a screen. (a) Destructive interference occurs here, because one path is a half wavelength longer than the other. The waves start in phase but arrive out of phase. (b) Constructive interference occurs here because one path is a whole wavelength longer than the other. The waves start out and arrive in phase.

[link] shows how to determine the path length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen (see the figure) is nearly the same for each path. The difference between the paths is shown in the figure; simple trigonometry shows it to be $d \sin \theta$, where d is the distance between the slits. To obtain **constructive interference for a double slit**, the path length difference must be an integral multiple of the wavelength, or

Equation:

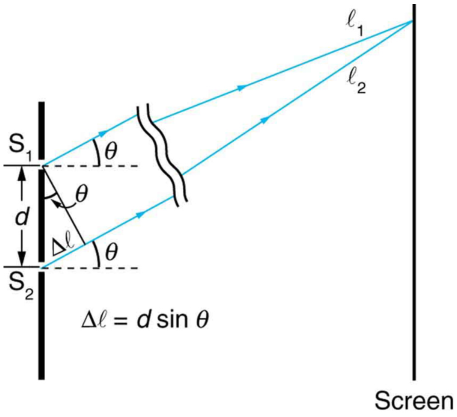
$$d \ \sin \theta = m\lambda, \ \text{for } m = 0, 1, \ -1, 2, \ -2, \ \ldots \ \ (\text{constructive}).$$

Similarly, to obtain **destructive interference for a double slit**, the path length difference must be a half-integral multiple of the wavelength, or

Equation:

$$d\ \sin\theta = \left(m + \frac{1}{2}\right)\lambda,\ \text{for}\ m = 0,\ 1,\ -1,\ 2,\ -2,\ \ldots\ \ (\text{destructive}),$$

where λ is the wavelength of the light, d is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call m the **order** of the interference. For example, $m = 4$ is fourth-order interference.



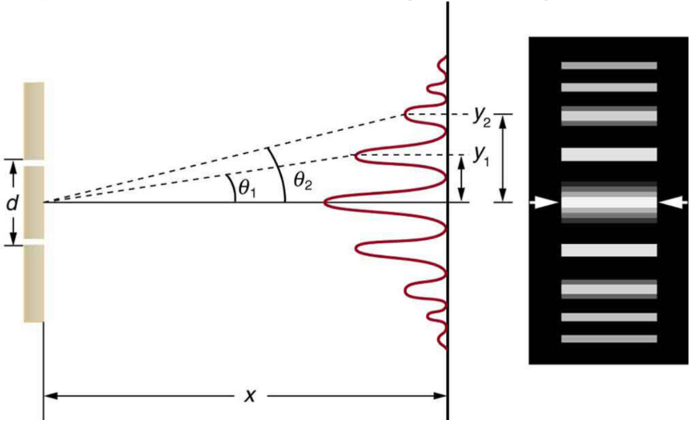
The paths from each slit to a common point on the screen differ by an amount d sin θ, assuming the distance to the screen is much greater than the distance between slits (not to scale here).

The equations for double slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on either side of the incident beam into a pattern called interference fringes, illustrated in [link]. The intensity of the bright fringes falls off on either side, being brightest at the center. The closer the slits are, the more is the spreading of the bright fringes. We can see this by examining the equation

Equation:

$$d\ \sin\theta = m\lambda\text{, for } m = 0, 1, -1, 2, -2, \ldots.$$

For fixed λ and m, the smaller d is, the larger θ must be, since $\sin\theta = m\lambda/d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large θ, hence a large effect.



The interference pattern for a double slit has an intensity that falls off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Example:

Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of 10.95° relative to the incident beam. What is the wavelength of the light?

Strategy

The third bright line is due to third-order constructive interference, which means that $m=3$. We are given $d=0.0100\text{ mm}$ and $\theta=10.95^{\circ}$. The wavelength can thus be found using the equation $d\ \sin\theta=m\lambda$ for constructive interference.

Solution

The equation is $d\ \sin\theta=m\lambda$. Solving for the wavelength λ gives

Equation:

$$\lambda=\frac{d\ \sin\theta}{m}.$$

Substituting known values yields

Equation:

$$\begin{aligned}\lambda &= \frac{(0.0100\text{ mm})(\sin 10.95^{\circ})}{3}\\ &= 6.33\times10^{-4}\text{ mm}=633\text{ nm}.\end{aligned}$$

Discussion

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ, so that spectra (measurements of intensity versus wavelength) can be obtained.

Example:
Calculating Highest Order Possible

Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

Strategy and Concept

The equation $d\ \sin\theta = m\lambda$ (for $m = 0, 1, -1, 2, -2, \ldots$) describes constructive interference. For fixed values of d and λ, the larger m is, the larger $\sin\theta$ is. However, the maximum value that $\sin\theta$ can have is 1, for an angle of 90º. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

Solution

Solving the equation $d\sin\theta = m\lambda$ for m gives

Equation:

$$m = \frac{d\sin\theta}{\lambda}.$$

Taking $\sin\theta = 1$ and substituting the values of d and λ from the preceding example gives

Equation:

$$m = \frac{(0.0100\ \text{mm})(1)}{633\ \text{nm}} \approx 15.8.$$

Therefore, the largest integer m can be is 15, or

Equation:

$$m = 15.$$

Discussion

The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.

Section Summary

- Young’s double slit experiment gave definitive proof of the wave character of light.
- An interference pattern is obtained by the superposition of light from two slits.
- There is constructive interference when $d\ \text{sin}\ \theta = m\lambda\ (\text{for}\ m = 0, 1,\ -1, 2,\ -2,\ \ldots)$, where d is the distance between the slits, θ is the angle relative to the incident direction, and m is the order of the interference.
- There is destructive interference when $d\ \ \text{sin}\ \theta = \left(m + \frac{1}{2}\right)\lambda\ (\text{for}\ \ m = 0, 1,\ -1, 2,\ -2,\ \ldots)$.

Conceptual Questions

Exercise:

Problem:

Young’s double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car? Explain.

Exercise:

Problem:

Suppose you use the same double slit to perform Young’s double slit experiment in air and then repeat the experiment in water. Do the angles to the same parts of the interference pattern get larger or smaller? Does the color of the light change? Explain.

Exercise:

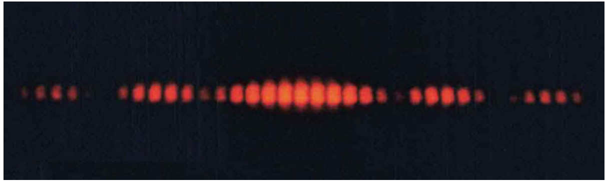
Problem:

Is it possible to create a situation in which there is only destructive interference? Explain.

Exercise:

Problem:

[link] shows the central part of the interference pattern for a pure wavelength of red light projected onto a double slit. The pattern is actually a combination of single slit and double slit interference. Note that the bright spots are evenly spaced. Is this a double slit or single slit characteristic? Note that some of the bright spots are dim on either side of the center. Is this a single slit or double slit characteristic? Which is smaller, the slit width or the separation between slits? Explain your responses.



This double slit interference pattern also shows signs of single slit interference. (credit: PASCO)

Problems & Exercises

Exercise:

Problem:

At what angle is the first-order maximum for 450-nm wavelength blue light falling on double slits separated by 0.0500 mm?

Solution:

0.516º

Exercise:

Problem:

Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.

Exercise:

Problem:

What is the separation between two slits for which 610-nm orange light has its first maximum at an angle of 30.0°?

Solution:

$1.22 \times 10^{-6}\ \text{m}$

Exercise:

Problem:

Find the distance between two slits that produces the first minimum for 410-nm violet light at an angle of 45.0°.

Exercise:

Problem:

Calculate the wavelength of light that has its third minimum at an angle of 30.0° when falling on double slits separated by $3.00\ \mu\text{m}$. Explicitly, show how you follow the steps in Problem-Solving Strategies for Wave Optics.

Solution:

600 nm

Exercise:

Problem:

What is the wavelength of light falling on double slits separated by $2.00\ \mu\text{m}$ if the third-order maximum is at an angle of 60.0°?

Exercise:

Problem:

At what angle is the fourth-order maximum for the situation in [link]?

Solution:

2.06º

Exercise:

Problem:

What is the highest-order maximum for 400-nm light falling on double slits separated by $25.0\ \mu m$?

Exercise:

Problem:

Find the largest wavelength of light falling on double slits separated by $1.20\ \mu m$ for which there is a first-order maximum. Is this in the visible part of the spectrum?

Solution:

1200 nm (not visible)

Exercise:

Problem:

What is the smallest separation between two slits that will produce a second-order maximum for 720-nm red light?

Exercise:

Problem:

(a) What is the smallest separation between two slits that will produce a second-order maximum for any visible light? (b) For all visible light?

Solution:

(a) 760 nm

(b) 1520 nm

Exercise:

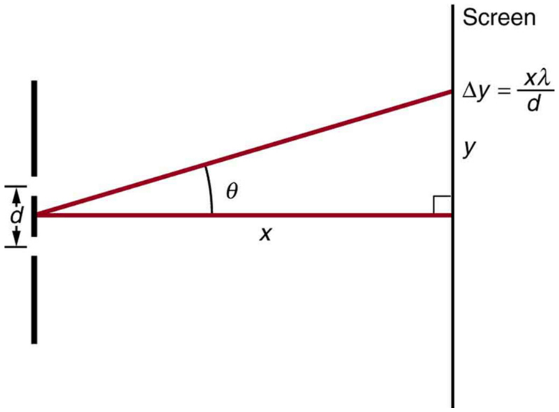
Problem:

(a) If the first-order maximum for pure-wavelength light falling on a double slit is at an angle of 10.0º, at what angle is the second-order maximum? (b) What is the angle of the first minimum? (c) What is the highest-order maximum possible here?

Exercise:

Problem:

[link] shows a double slit located a distance x from a screen, with the distance from the center of the screen given by y. When the distance d between the slits is relatively large, there will be numerous bright spots, called fringes. Show that, for small angles (where $\sin\theta \approx \theta$, with θ in radians), the distance between fringes is given by $\Delta y = x\lambda / d$.



The distance between adjacent fringes is $\Delta y = x\lambda / d$, assuming the

slit separation d is large
compared with λ.

Solution:

For small angles $\sin\,\theta - \tan\,\theta \approx \theta$ (in radians).

For two adjacent fringes we have,

Equation:

$$d\ \ \sin\,\theta_{\text{m}} = m\lambda$$

and

Equation:

$$d\ \ \sin\,\theta_{\text{m}+1} = (m+1)\lambda$$

Subtracting these equations gives

Equation:

$$\begin{gathered} d(\sin\,\theta_{\text{m}+1} - \sin\,\theta_{\text{m}}) = [(m+1) - m]\lambda \\ d(\theta_{\text{m}+1} {-} \theta_{\text{m}}) = \lambda \\ \tan\,\theta_{\text{m}} = \frac{y_{\text{m}}}{x} \approx \theta_{\text{m}} \Rightarrow d\left(\frac{y_{\text{m}+1}}{x} - \frac{y_{\text{m}}}{x}\right) = \lambda \\ d\frac{\Delta y}{x} = \lambda \Rightarrow \Delta y = \frac{x\lambda}{d} \end{gathered}$$

Exercise:

Problem:

Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in [link].

Exercise:

Problem:

Using the result of the problem two problems prior, find the wavelength of light that produces fringes 7.50 mm apart on a screen 2.00 m from double slits separated by 0.120 mm (see [link]).

Solution:

450 nm

Glossary

coherent
    waves are in phase or have a definite phase relationship

constructive interference for a double slit
    the path length difference must be an integral multiple of the wavelength

destructive interference for a double slit
    the path length difference must be a half-integral multiple of the wavelength

incoherent
    waves have random phase relationships

order
    the integer m used in the equations for constructive and destructive interference for a double slit

Superposition and Interference

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.

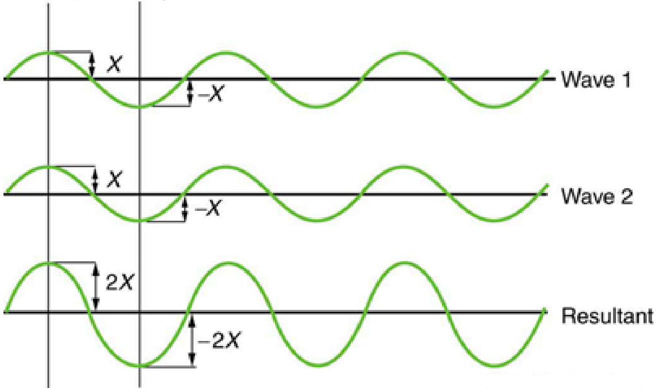
These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

Most waves do not look very simple. They look more like the waves in [link] than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

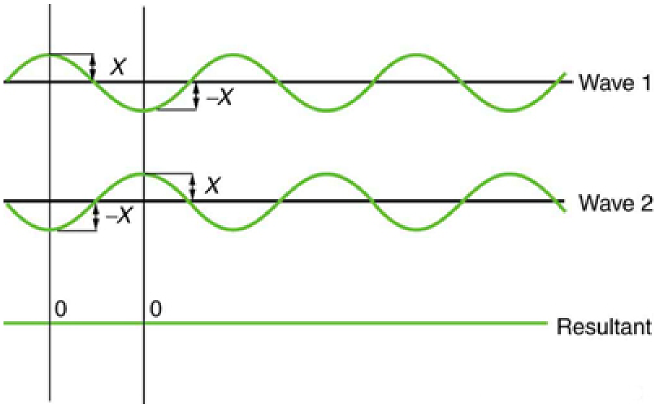
When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. [link] and [link] illustrate superposition in two special cases, both of which produce simple results.

[link] shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[link] shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



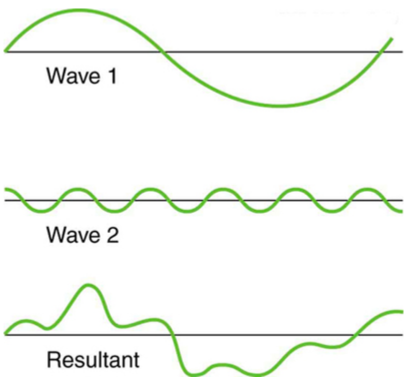
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [link]. Here again, the disturbances add and subtract, producing a more complicated looking wave.

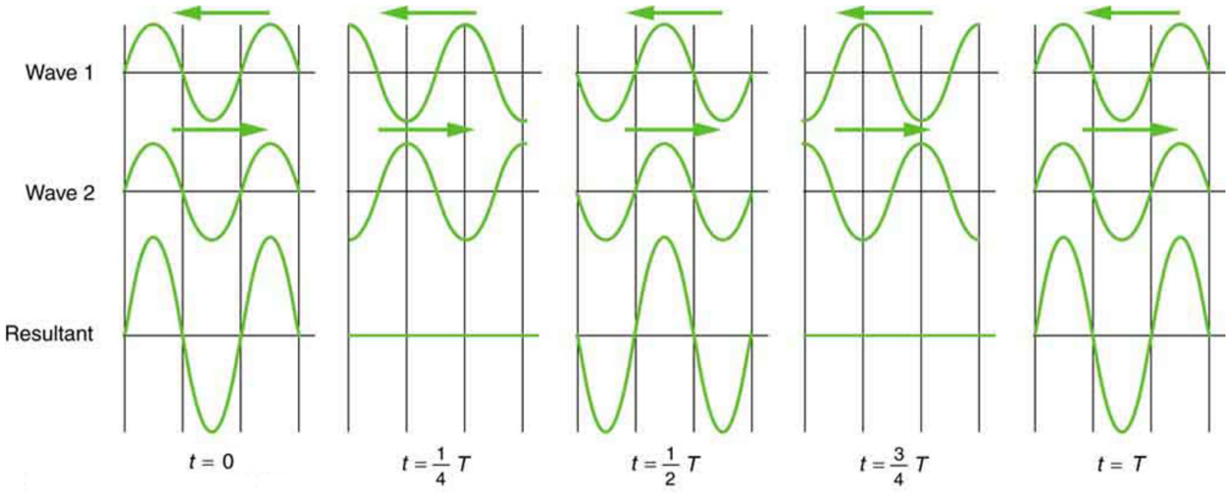


Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in [link] for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

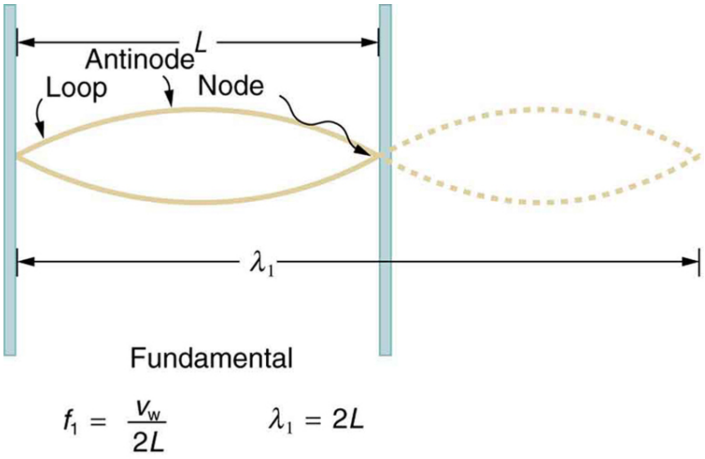
A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.



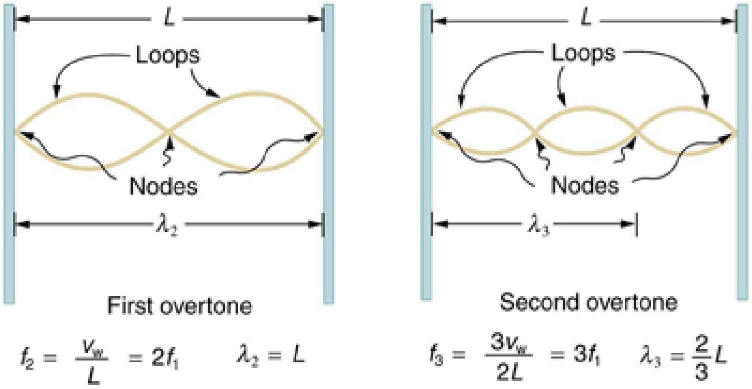
Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. [link] and [link] show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_w/\lambda_1 = v_w/2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in [link], the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_w/\lambda_2 = v_w/2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.



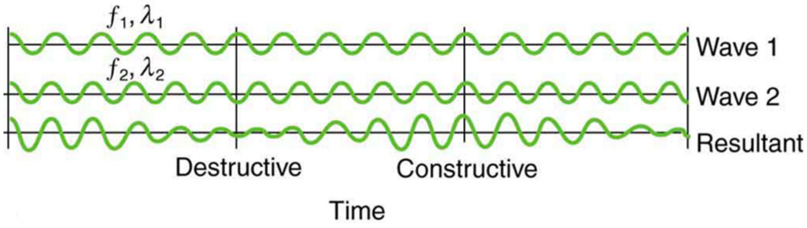
The figure shows a string oscillating at its fundamental frequency.


L
Loops
Nodes
λ_2
First overtone
$f_2 = \frac{v_w}{L} = 2f_1$
$\lambda_2 = L$
L
Loops
Nodes
λ_3
Second overtone
$f_3 = \frac{3v_w}{2L} = 3f_1$
$\lambda_3 = \frac{2}{3}L$

First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. [link] illustrates this graphically.



Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

Equation:

$$x = X\cos\left(\frac{2\pi\, t}{T}\right) = X\cos(2\pi\ \mathrm{ft}),$$

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

Equation:

$$x = x_1 + x_2.$$

More specifically,

Equation:

$$x = X\cos(2\pi\ f_1 t) + X\cos(2\pi\ f_2 t).$$

Using a trigonometric identity, it can be shown that

Equation:

$$x = 2X\cos(\pi f_{\text{B}}t)\cos(2\pi f_{\text{ave}}t),$$

where

Equation:

$$f_{\text{B}} = \mid f_1 - f_2 \mid$$

is the beat frequency, and f_{ave} is the average of f_1 and f_2. These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_{B}. The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave}. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Note:

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Exercise:

Check Your Understanding

Problem:

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution:

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Exercise:

Check Your Understanding

Problem: Define nodes and antinodes.

Solution:

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Exercise:

Check Your Understanding

Problem:

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution:

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

Note:
PhET Explorations: Wave Interference
Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

Wave Interference

Section Summary

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.

- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by
  Equation:

$$f_{\mathrm{B}} = \mid f_1 - f_2 \mid .$$

Conceptual Questions

Exercise:

Problem:

Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

Problems & Exercises

Exercise:

Problem:

A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

Solution:

$f = 4 \text{ Hz}$

Exercise:

Problem:

The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

Exercise:

Problem:

Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

Solution:

462 Hz,

4 Hz

Exercise:

Problem:

Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

Exercise:

Problem:

A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

Solution:

(a) 3.33 m/s

(b) 1.25 Hz

Exercise:

Problem:

Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

Glossary

antinode
the location of maximum amplitude in standing waves

beat frequency
the frequency of the amplitude fluctuations of a wave

constructive interference
when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference
when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

fundamental frequency
the lowest frequency of a periodic waveform

nodes
the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

overtones

multiples of the fundamental frequency of a sound

superposition

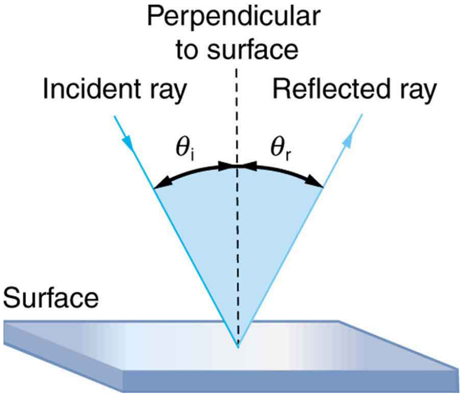
the phenomenon that occurs when two or more waves arrive at the same point

The Law of Reflection

- Explain reflection of light from polished and rough surfaces.

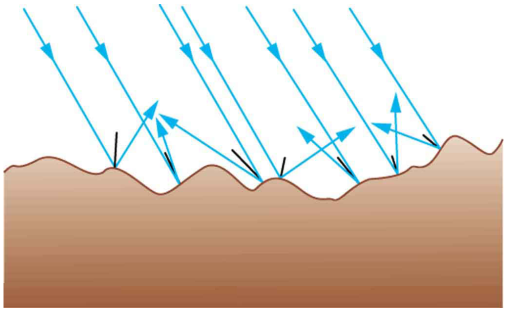
Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

The law of reflection is illustrated in [link], which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but [link] illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in [link]. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in [link]. When the moon reflects from a lake, as shown in [link], a combination of these effects takes place.

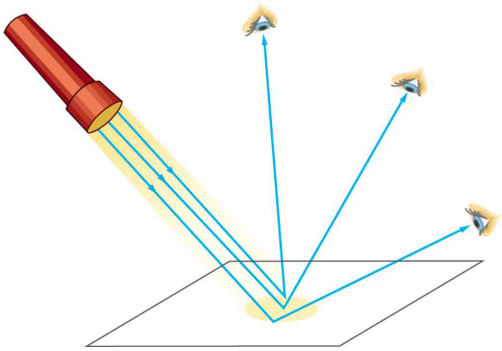


The law of reflection states that the angle of reflection equals the angle of incidence—$\theta_r = \theta_i$. The angles are measured relative to the perpendicular to

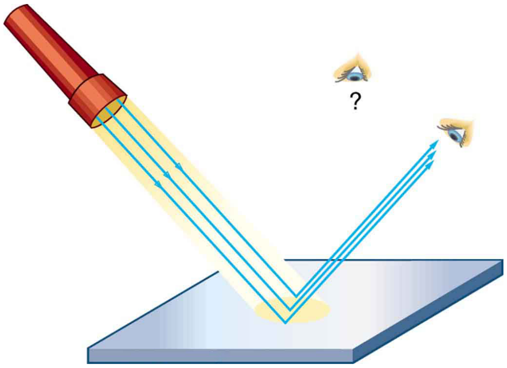
the surface at the point
where the ray strikes
the surface.



Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.



When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because its surface is rough and diffuses the light.



A mirror illuminated by many parallel rays reflects them in only one direction, since its surface is very smooth. Only the observer at a particular angle will see the reflected light.

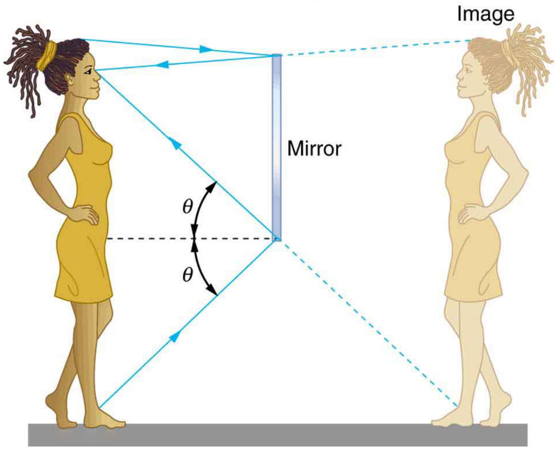
Moonlight is spread out when it is reflected by the lake, since the surface is shiny but uneven. (credit:

Diego Torres Silvestre,
Flickr)

The law of reflection is very simple: The angle of reflection equals the angle of incidence.

Note:
The Law of Reflection
The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in [link]. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.



Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

Note:
Take-Home Experiment: Law of Reflection

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in [link]. Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in [link] and [link]? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

Section Summary

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

Conceptual Questions

Exercise:

Problem:

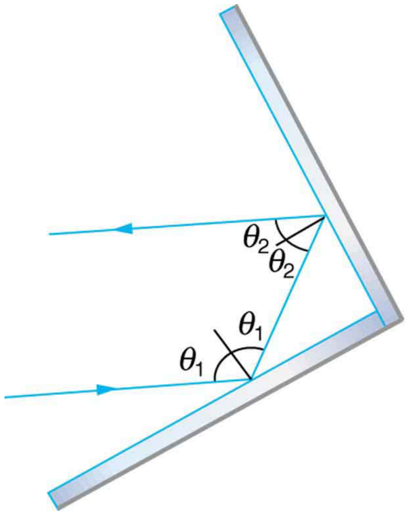
Using the law of reflection, explain how powder takes the shine off of a person’s nose. What is the name of the optical effect?

Problems & Exercises

Exercise:

Problem:

Show that when light reflects from two mirrors that meet each other at a right angle, the outgoing ray is parallel to the incoming ray, as illustrated in the following figure.



A corner reflector sends the reflected ray back in a direction parallel to the incident ray, independent of incoming direction.

Exercise:

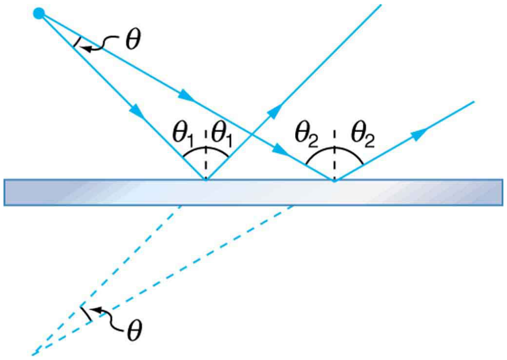
Problem:

Light shows staged with lasers use moving mirrors to swing beams and create colorful effects. Show that a light ray reflected from a mirror changes direction by 2θ when the mirror is rotated by an angle θ.

Exercise:

Problem:

A flat mirror is neither converging nor diverging. To prove this, consider two rays originating from the same point and diverging at an angle θ. Show that after striking a plane mirror, the angle between their directions remains θ.



A flat mirror neither converges nor diverges light rays. Two rays continue to diverge at the same angle after reflection.

Glossary

mirror

smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

law of reflection
angle of reflection equals the angle of incidence

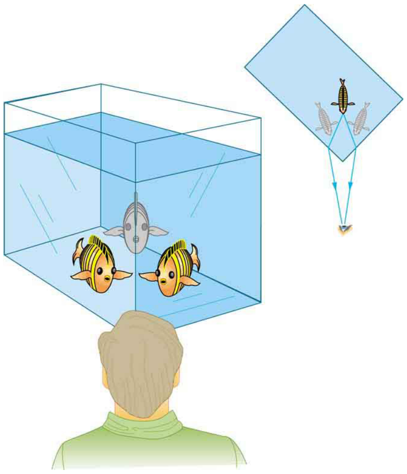
The Law of Refraction

- Determine the index of refraction, given the speed of light in a medium.

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [link].) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray’s direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Note:
Refraction
The changing of a light ray’s direction (loosely called bending) when it passes through variations in matter is called refraction.

Note:
Speed of Light
The speed of light c not only affects refraction, it is one of the central concepts of Einstein’s theory of relativity. As the accuracy of the measurements of the speed of light were improved, c was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in Special Relativity. It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.

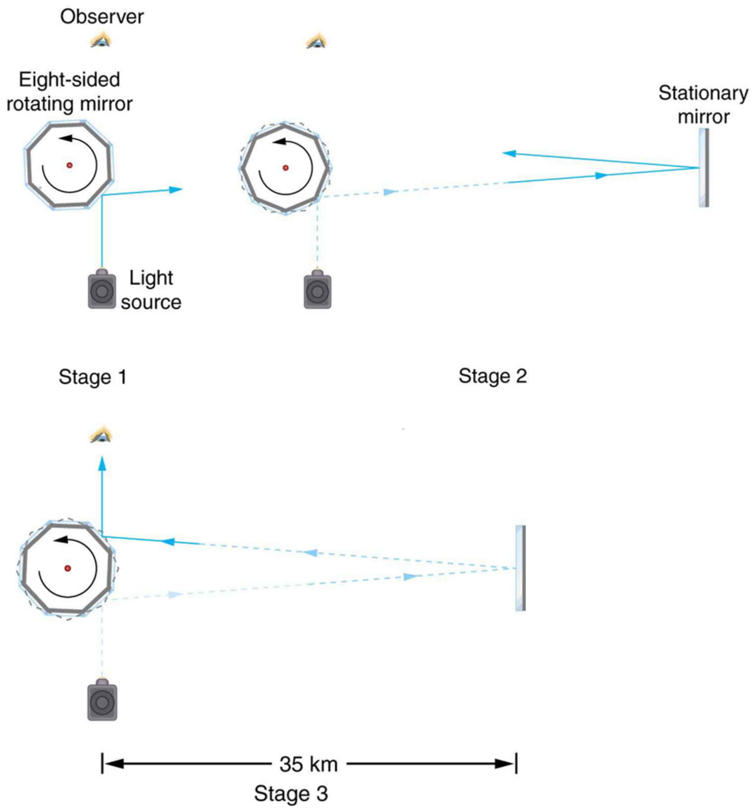


Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter’s moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be $2.26 \times 10^8\ \mathrm{m/s}$ (only 25% different from today’s accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [link]. Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer’s eye.


Observer
Eight-sided rotating mirror
Stationary mirror
Light source
Stage 1
Stage 2
35 km
Stage 3

A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson’s calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

Equation:

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s},$$

where the approximate value of $3.00 \times 10^8 \text{ m/s}$ is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

Equation:

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Note:

Value of the Speed of Light

Equation:

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$$

Note:

Index of Refraction

Equation:

$$n = \frac{c}{v}$$

That is, $n \geq 1$. [link] gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n = 1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Medium	***n***
Gases at 0°C***, 1 atm***	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at 20°C	
Benzene	1.501
Carbon disulfide	1.628

Medium	***n***
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at 20°C	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 20°C	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Index of Refraction in Various Media

Example:

Speed of Light in Matter

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Strategy

The speed of light in a material, v, can be calculated from the index of refraction n of the material using the equation $n = c/v$.

Solution

The equation for index of refraction states that $n = c/v$. Rearranging this to determine v gives

Equation:

$$v = \frac{c}{n}.$$

The index of refraction for zircon is given as 1.923 in [link], and c is given in the equation for speed of light. Entering these values in the last expression gives

Equation:

$$\begin{aligned} v &= \frac{3.00 \times 10^8 \text{ m/s}}{1.923} \\ &= 1.56 \times 10^8 \text{ m/s}. \end{aligned}$$

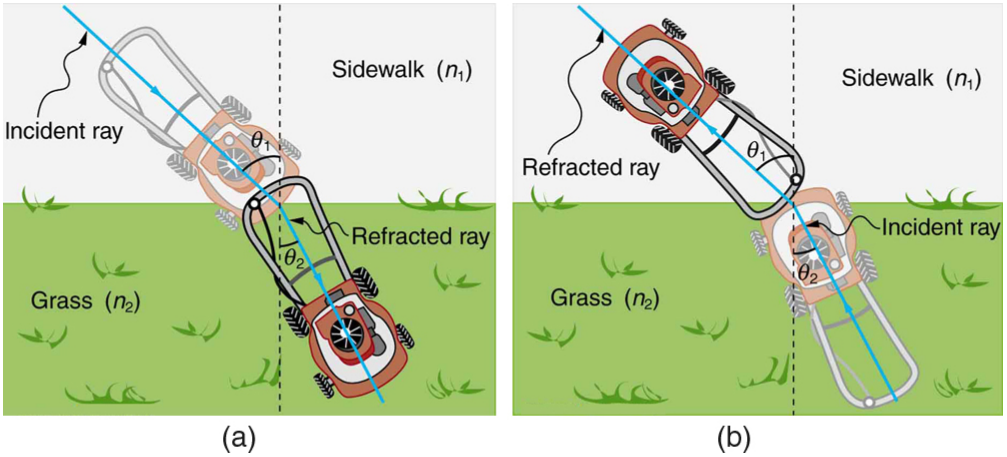
Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [link] that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

[link] shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it.

(Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in [link], medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in [link](a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in [link](b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.



The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the **law of refraction**, or "Snell's Law," which is stated in equation form as

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and θ_2 are the angles between the rays and the perpendicular in medium 1 and 2, as shown in [link]. The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell's law after the Dutch mathematician Willebrord Snell (1591–1626), who discovered it in 1621. Snell's experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium. Snell was not aware that the speed of light varied in different media, but through experiments he was able to determine indices of refraction from the way light rays changed direction.

Note:

The Law of Refraction

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Note:

Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

Example:

Determine the Index of Refraction from Refraction Data

Find the index of refraction for medium 2 in [link](a), assuming medium 1 is air and given the incident angle is 30.0º and the angle of refraction is 22.0º.

Strategy

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus $n_1 = 1.00$ here. From the given information, $\theta_1 = 30.0^\text{º}$ and $\theta_2 = 22.0^\text{º}$. With this information, the only unknown in Snell's law is n_2, so that it can be used to find this unknown.

Solution

Snell's law is

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Rearranging to isolate n_2 gives

Equation:

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2}.$$

Entering known values,

Equation:

$$\begin{aligned} n_2 &= 1.00 \frac{\sin 30.0^\text{º}}{\sin 22.0^\text{º}} = \frac{0.500}{0.375} \\ &= 1.33. \end{aligned}$$

Discussion

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.

Example:

A Larger Change in Direction

Suppose that in a situation like that in [link], light goes from air to diamond and that the incident angle is 30.0º. Calculate the angle of refraction θ_2 in the diamond.

Strategy

Again the index of refraction for air is taken to be $n_1 = 1.00$, and we are given $\theta_1 = 30.0^{\circ}$. We can look up the index of refraction for diamond in [link], finding $n_2 = 2.419$. The only unknown in Snell's law is θ_2, which we wish to determine.

Solution

Solving Snell's law for sin θ_2 yields

Equation:

$$\sin\,\theta_2 = \frac{n_1}{n_2}\sin\,\theta_1.$$

Entering known values,

Equation:

$$\sin\,\theta_2 = \frac{1.00}{2.419}\sin\,30.0^{\circ} = \left(0.413\right)(0.500) = 0.207.$$

The angle is thus

Equation:

$$\theta_2 = \sin^{-1}0.207 = 11.9^{\circ}.$$

Discussion

For the same 30° angle of incidence, the angle of refraction in diamond is significantly smaller than in water (11.9° rather than 22°—see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

Section Summary

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum $c=2.99792458\times 10^{8}\ \text{m/s}\approx 3.00\times 10^{8}\ \text{m/s}$.
- Index of refraction $n=\frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as $n_1 \sin \theta_1 = n_2 \sin \theta_2$.

Conceptual Questions

Exercise:

Problem:

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

Exercise:

Problem:

Why is the index of refraction always greater than or equal to 1?

Exercise:

Problem:

Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

Exercise:

Problem:

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

Exercise:

Problem:

Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

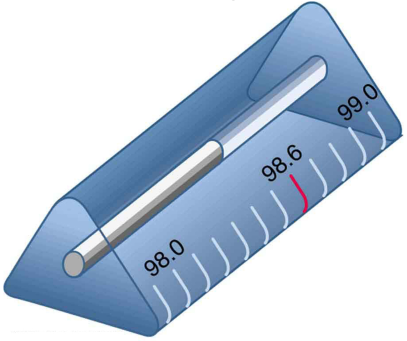
Exercise:

Problem:

Explain why a person’s legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

Exercise:

Problem: Why is the front surface of a thermometer curved as shown?



The curved surface
of the thermometer
serves a purpose.

Exercise:

Problem:

Suppose light were incident from air onto a material that had a negative index of refraction, say –1.3; where does the refracted light ray go?

Problems & Exercises

Exercise:

Problem: What is the speed of light in water? In glycerine?

Solution:

$2.25 \times 10^8 \text{ m/s}$ in water

$2.04 \times 10^8 \text{ m/s}$ in glycerine

Exercise:

Problem: What is the speed of light in air? In crown glass?

Exercise:

Problem:

Calculate the index of refraction for a medium in which the speed of light is $2.012 \times 10^8 \text{ m/s}$, and identify the most likely substance based on [link].

Solution:

1.490, polystyrene

Exercise:

Problem:

In what substance in [link] is the speed of light $2.290 \times 10^8 \text{ m/s}$?

Exercise:

Problem:

There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is $3.84 \times 10^5 \text{ km}$ away, would the light first arrive on Earth?

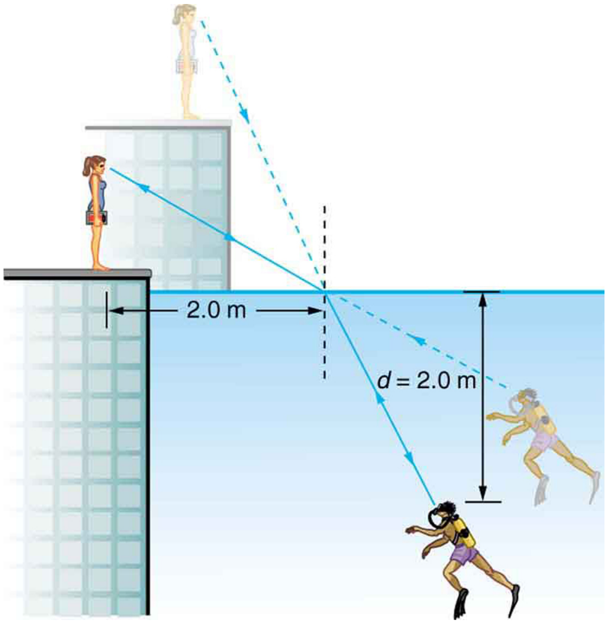
Solution:

1.28 s

Exercise:

Problem:

A scuba diver training in a pool looks at his instructor as shown in [link]. What angle does the ray from the instructor’s face make with the perpendicular to the water at the point where the ray enters? The angle between the ray in the water and the perpendicular to the water is 25.0°.



A scuba diver in a pool and his trainer look at each other.

Exercise:

Problem:

Components of some computers communicate with each other through optical fibers having an index of refraction $n = 1.55$. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

Solution:

1.03 ns

Exercise:

Problem:

(a) Given that the angle between the ray in the water and the perpendicular to the water is 25.0^{o}, and using information in [link], find the height of the instructor’s head above the water, noting that you will first have to calculate the angle of incidence. (b) Find the apparent depth of the diver’s head below water as seen by the instructor.

Exercise:

Problem:

Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of 45.0^{o}, and you observe the angle of refraction to be 40.3^{o}. What is the index of refraction of the substance and its likely identity?

Solution:

$n = 1.46$, fused quartz

Exercise:

Problem:

On the Moon’s surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth’s atmosphere? Assume the distance to the Moon is precisely $3.84 \times 10^{8}\ \mathrm{m}$, and Earth’s atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction $n = 1.000293$.

Exercise:

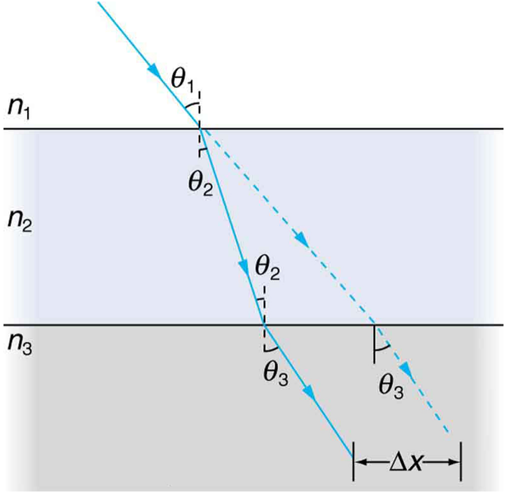
Problem:

Suppose [link] represents a ray of light going from air through crown glass into water, such as going into a fish tank. Calculate the amount the ray is displaced by the glass (Δx), given that the incident angle is 40.0º and the glass is 1.00 cm thick.

Exercise:

Problem:

[link] shows a ray of light passing from one medium into a second and then a third. Show that θ_3 is the same as it would be if the second medium were not present (provided total internal reflection does not occur).



A ray of light passes from one medium to a third by traveling through a second. The final direction is the same as if the second medium were not present, but the ray is displaced by Δx (shown exaggerated).

Exercise:

Problem: **Unreasonable Results**

Suppose light travels from water to another substance, with an angle of incidence of 10.0^{o} and an angle of refraction of 14.9^{o}. (a) What is the index of refraction of the other substance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 0.898

(b) Can’t have $n < 1.00$ since this would imply a speed greater than c.

(c) Refracted angle is too big relative to the angle of incidence.

Exercise:

Problem: **Construct Your Own Problem**

Consider sunlight entering the Earth’s atmosphere at sunrise and sunset—that is, at a 90^{o} incident angle. Taking the boundary between nearly empty space and the atmosphere to be sudden, calculate the angle of refraction for sunlight. This lengthens the time the Sun appears to be above the horizon, both at sunrise and sunset. Now construct a problem in which you determine the angle of refraction for different models of the atmosphere, such as various layers of varying density. Your instructor may wish to guide you on the level of complexity to consider and on how the index of refraction varies with air density.

Exercise:

Problem: **Unreasonable Results**

Light traveling from water to a gemstone strikes the surface at an angle of 80.0^{o} and has an angle of refraction of 15.2^{o}. (a) What is the speed of light in the gemstone? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $\frac{c}{5.00}$

(b) Speed of light too slow, since index is much greater than that of diamond.

(c) Angle of refraction is unreasonable relative to the angle of incidence.

Glossary

refraction	changing of a light ray’s direction when it passes through variations in matter
index of refraction	for a material, the ratio of the speed of light in vacuum to that in the material

Selected Radioactive Isotopes

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Isotope	***t* 1/2**	**DecayMode(s)**	**Energy(MeV)**	**Percent**		**γ-Ray Energy(MeV)**
^{3}H	12.33 y	β^-	0.0186	100%		
^{14}C	5730 y	β^-	0.156	100%		
^{13}N	9.96 min	β^+	1.20	100%		
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27
^{32}P	14.28 d	β^-	1.71	100%		
^{35}S	87.4 d	β^-	0.167	100%		
^{36}Cl	3.00×10^5y	β^-	0.710	100%		
^{40}K	1.28×10^9y	β^-	1.31	89%		
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373
						0.618
^{45}Ca	165 d	β^-	0.257	100%		
^{51}Cr	27.70 d	EC			γ	0.320
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33
						1.43
^{52}Fe	8.27 h	β^+	1.80	43%		0.169
						0.378
^{59}Fe	44.6 d	β^-s	0.273	45%	γs	1.10
			0.466	55%		1.29
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17
						1.33
^{65}Zn	244.1 d	EC			γ	1.12

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ-Ray Energy(MeV)
^{67}Ga	78.3 h	EC			γs	0.0933
						0.185
						0.300
						others
^{75}Se	118.5 d	EC			γs	0.121
						0.136
						0.265
						0.280
						others
^{86}Rb	18.8 d	β^-s	0.69	9%	γ	1.08
			1.77	91%		
^{85}Sr	64.8 d	EC			γ	0.514
^{90}Sr	28.8 y	β^-	0.546	100%		
^{90}Y	64.1 h	β^-	2.28	100%		
$^{99\text{m}}\text{Tc}$	6.02 h	IT			γ	0.142
$^{113\text{m}}\text{In}$	99.5 min	IT			γ	0.392
^{123}I	13.0 h	EC			γ	0.159
^{131}I	8.040 d	β^-s	0.248	7%	γs	0.364
			0.607	93%		others
			others			
^{129}Cs	32.3 h	EC			γs	0.0400
						0.372
						0.411
						others
^{137}Cs	30.17 y	β^-s	0.511	95%	γ	0.662
			1.17	5%		

Isotope	***t* 1/2**	**DecayMode(s)**	**Energy(MeV)**	**Percent**		**γ-Ray Energy(MeV)**
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γs	0.030
						0.044
						0.537
						others
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412
^{197}Hg	64.1 h	EC			γ	0.0733
^{210}Po	138.38 d	α	5.41	100%		
^{226}Ra	1.60×10^3y	αs	4.68	5%	γ	0.186
			4.87	95%		
^{235}U	7.038×10^8y	α	4.68	$\approx 100\%$	γs	numerous
^{238}U	4.468×10^9y	αs	4.22	23%	γ	0.050
			4.27	77%		
^{237}Np	2.14×10^6y	αs	numerous		γs	numerous
			4.96 (max.)			
^{239}Pu	2.41×10^4y	αs	5.19	11%	γs	7.5×10^{-5}
			5.23	15%		0.013
			5.24	73%		0.052
						others
^{243}Am	7.37×10^3y	αs	Max. 5.44		γs	0.075
			5.37	88%		others
			5.32	11%		
			others			

Selected Radioactive Isotopes

Useful Information

This appendix is broken into several tables.

- [link], Important Constants
- [link], Submicroscopic Masses
- [link], Solar System Data
- [link], Metric Prefixes for Powers of Ten and Their Symbols
- [link], The Greek Alphabet
- [link], SI units
- [link], Selected British Units
- [link], Other Units
- [link], Useful Formulae

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^{8}\,\text{m/s}$	$3.00 \times 10^{8}\,\text{m/s}$
G	Gravitational constant	$6.67408(31) \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$
N_A	Avogadro’s number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann’s constant	$1.3806488(13) \times 10^{-23}\,\text{J/K}$	$1.38 \times 10^{-23}\,\text{J/K}$
R	Gas constant	$8.3144621(75)\,\text{J/mol}\cdot\text{K}$	$8.31\,\text{J/mol}\cdot\text{K} = 1.99\,\text{cal/mol}\cdot\text{K} =$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8}\,\text{W/m}^2\cdot\text{K}$	$5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}$
k	Coulomb force constant	$8.987551788... \times 10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2$	$8.99 \times 10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19}\,\text{C}$	$-1.60 \times 10^{-19}\,\text{C}$
ε_0	Permittivity of free space	$8.854187817... \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$	$8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$
μ_0	Permeability of free space	$4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$	$1.26 \times 10^{-6}\,\text{T}\cdot\text{m/A}$
h	Planck’s constant	$6.62606957(29) \times 10^{-34}\,\text{J}\cdot\text{s}$	$6.63 \times 10^{-34}\,\text{J}\cdot\text{s}$

Important Constants[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, an
www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Nu
are exact as defined.

Symbol	**Meaning**	**Best Value**	**Approximate Value**
m_e	Electron mass	$9.10938291(40)\times10^{-31}\text{kg}$	$9.11\times10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74)\times10^{-27}\text{kg}$	$1.6726\times10^{-27}\text{kg}$
m_n	Neutron mass	$1.674927351(74)\times10^{-27}\text{kg}$	$1.6749\times10^{-27}\text{kg}$
u	Atomic mass unit	$1.660538921(73)\times10^{-27}\text{kg}$	$1.6605\times10^{-27}\text{kg}$

Submicroscopic Masses[footnote]
Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Sun	mass	$1.99\times10^{30}\text{kg}$
	average radius	$6.96\times10^{8}\text{m}$
	Earth-sun distance (average)	$1.496\times10^{11}\text{m}$
Earth	mass	$5.9736\times10^{24}\text{kg}$
	average radius	$6.376\times10^{6}\text{m}$
	orbital period	$3.16\times10^{7}\text{s}$

Moon	mass	$7.35 \times 10^{22} \text{kg}$
	average radius	$1.74 \times 10^{6} \text{m}$
	orbital period (average)	$2.36 \times 10^{6} \text{s}$
	Earth-moon distance (average)	$3.84 \times 10^{8} \text{m}$

Solar System Data

Prefix	**Symbol**	**Value**	**Prefix**	**Symbol**	**Value**
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0} (= 1)$	femto	f	10^{-15}

Metric Prefixes for Powers of Ten and Their Symbols

Alpha	A	α	Eta	H	η	Nu	N	ν	Tau	T	τ
Beta	B	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	I	ι	Omicron	O	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	K	κ	Pi	Π	π	Chi	X	χ

Epsilon	E	ε	Lambda	Λ	λ	Rho	P	ρ	Psi	Ψ	ψ
Zeta	Z	ζ	Mu	M	μ	Sigma	Σ	σ	Omega	Ω	ω

The Greek Alphabet

	Entity	**Abbreviation**	**Name**
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$\mathrm{N} = \mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2$	newton
	Energy	$\mathrm{J} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$	joule
	Power	$\mathrm{W} = \mathrm{J}/\mathrm{s}$	watt
	Pressure	$\mathrm{Pa} = \mathrm{N}/\mathrm{m}^2$	pascal
	Frequency	$\mathrm{Hz} = 1/\mathrm{s}$	hertz
	Electronic potential	$\mathrm{V} = \mathrm{J}/\mathrm{C}$	volt
	Capacitance	$\mathrm{F} = \mathrm{C}/\mathrm{V}$	farad
	Charge	$\mathrm{C} = \mathrm{s} \cdot \mathrm{A}$	coulomb
	Resistance	$\Omega = \mathrm{V}/\mathrm{A}$	ohm

	Entity	**Abbreviation**	**Name**
	Magnetic field	$\mathrm{T} = \mathrm{N}/(\mathrm{A} \cdot \mathrm{m})$	tesla
	Nuclear decay rate	$\mathrm{Bq} = 1/\mathrm{s}$	becquerel

SI Units

Length	$1\ \text{inch (in.)} = 2.54\ \text{cm (exactly)}$
	$1\ \text{foot (ft)} = 0.3048\ \text{m}$
	$1\ \text{mile (mi)} = 1.609\ \text{km}$
Force	$1\ \text{pound (lb)} = 4.448\ \text{N}$
Energy	$1\ \text{British thermal unit (Btu)} = 1.055 \times 10^3\ \text{J}$
Power	$1\ \text{horsepower (hp)} = 746\ \text{W}$
Pressure	$1\ \text{lb/in}^2 = 6.895 \times 10^3\ \text{Pa}$

Selected British Units

Length	$1\ \text{light year (ly)} = 9.46 \times 10^{15}\ \text{m}$
	$1\ \text{astronomical unit (au)} = 1.50 \times 10^{11}\ \text{m}$
	$1\ \text{nautical mile} = 1.852\ \text{km}$
	$1\ \text{angstrom(Å)}\ = 10^{-10}\ \text{m}$
Area	$1\ \text{acre (ac)} = 4.05 \times 10^3\ \text{m}^2$
	$1\ \text{square foot (ft}^2) = 9.29 \times 10^{-2}\ \text{m}^2$
	$1\ \text{barn}\ (b) = 10^{-28}\ \text{m}^2$
Volume	$1\ \text{liter}\ (L) = 10^{-3}\ \text{m}^3$

	1 U.S. gallon (gal) $= 3.785 \times 10^{-3}\,\mathrm{m}^3$
Mass	1 solar mass $= 1.99 \times 10^{30}\,\mathrm{kg}$
	1 metric ton $= 10^3\,\mathrm{kg}$
	1 atomic mass unit $(u) = 1.6605 \times 10^{-27}\,\mathrm{kg}$
Time	1 year $(y) = 3.16 \times 10^7\,\mathrm{s}$
	1 day $(d) = 86\ 400\,\mathrm{s}$
Speed	1 mile per hour $(\mathrm{mph}) = 1.609\,\mathrm{km/h}$
	1 nautical mile per hour $(\mathrm{naut}) = 1.852\,\mathrm{km/h}$
Angle	1 degree $(^\circ) = 1.745 \times 10^{-2}\,\mathrm{rad}$
	1 minute of arc $(') = 1/60$ degree
	1 second of arc $('') = 1/60$ minute of arc
	1 grad $= 1.571 \times 10^{-2}\,\mathrm{rad}$
Energy	1 kiloton TNT $(\mathrm{kT}) = 4.2 \times 10^{12}\,\mathrm{J}$
	1 kilowatt hour $(\mathrm{kW} \cdot h) = 3.60 \times 10^6\,\mathrm{J}$
	1 food calorie $(\mathrm{kcal}) = 4186\,\mathrm{J}$
	1 calorie $(\mathrm{cal}) = 4.186\,\mathrm{J}$
	1 electron volt $(\mathrm{eV}) = 1.60 \times 10^{-19}\,\mathrm{J}$
Pressure	1 atmosphere $(\mathrm{atm}) = 1.013 \times 10^5\,\mathrm{Pa}$
	1 millimeter of mercury $(\mathrm{mm\,Hg}) = 133.3\,\mathrm{Pa}$
	1 torricelli $(\mathrm{torr}) = 1\,\mathrm{mm\,Hg} = 133.3\,\mathrm{Pa}$
Nuclear decay rate	1 curie $(\mathrm{Ci}) = 3.70 \times 10^{10}\,\mathrm{Bq}$

Other Units

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$

Volume of a sphere with radius r	$V = (4/3)\ \pi r^3$

Useful Formulae

Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	Definition
any symbol	average (indicated by a bar over a symbol—e.g., v is average velocity)
$^\circ\text{C}$	Celsius degree
$^\circ\text{F}$	Fahrenheit degree
$//$	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus

Symbol	Definition
$_0$	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^-	electron emitted in nuclear beta decay
β^+	positron decay
γ	gamma rays

Symbol	Definition
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
$\varDelta E$	change in energy between the initial and final orbits of an electron in an atom
$\varDelta E$	uncertainty in energy
$\varDelta m$	difference in mass between initial and final products
$\varDelta N$	number of decays that occur
$\varDelta p$	change in momentum

Symbol	Definition
Δp	uncertainty in momentum
$\Delta\mathrm{PE}_{\mathrm{g}}$	change in gravitational potential energy
$\Delta\theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference
Δx	uncertainty in position
ε_0	permittivity of free space
η	viscosity

Symbol	**Definition**
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium

Symbol	**Definition**
μ_0	permeability of free space
μ_{k}	coefficient of kinetic friction
μ_{s}	coefficient of static friction
v_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_{c}	critical density, the density needed to just halt universal expansion
ρ_{fl}	fluid density

Symbol	Definition
ρ_{obj}	average density of an object
ρ/ρ_{w}	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity

Symbol	**Definition**
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_{B}	Bohr radius
a_{c}	centripetal acceleration
a_{t}	tangential acceleration
AC	alternating current
AM	amplitude modulation

Symbol	**Definition**
atm	atmosphere
B	baryon number
B	blue quark color
B	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
$\mathrm{B}_{\mathrm{int}}$	electron’s intrinsic magnetic field
$\mathrm{B}_{\mathrm{orb}}$	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons

Symbol	Definition
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_p	total capacitance in parallel
C_s	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat

Symbol	Definition
c	speed of light
Cal	kilocalorie
cal	calorie
COP_{hp}	heat pump’s coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos\theta$	cosine
$\cot\theta$	cotangent
$\csc\theta$	cosecant
D	diffusion constant
d	displacement

Symbol	**Definition**
d	quark flavor down
dB	decibel
d_{i}	distance of an image from the center of a lens
d_{o}	distance of an object from the center of a lens
DC	direct current
E	electric field strength
ε	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy

Symbol	Definition
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
$\mathrm{Eff}_{\mathrm{C}}$	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor

Symbol	Definition
E_{out}	energy output
e	emissivity of an object
e^{+}	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
$\mathbf{F}$	force
F	magnitude of a force
F	restoring force
F_{B}	buoyant force

Symbol	**Definition**
F_c	centripetal force
F_i	force input
$\mathbf{F}_\mathrm{net}$	net force
F_o	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)

Symbol	Definition
f_1	fundamental
f_2	first overtone
f_3	second overtone
f_B	beat frequency
f_k	magnitude of kinetic friction
f_s	magnitude of static friction
G	gravitational constant
G	green quark color
G	antigreen (magenta) antiquark color

Symbol	Definition
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
h_i	height of the image
h_o	height of the object
I	electric current

Symbol	**Definition**
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J/Ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant

Symbol	**Definition**
k	force constant of a spring
K_α	x rays created when an electron falls into an $n=1$ shell vacancy from the $n=3$ shell
K_β	x rays created when an electron falls into an $n=2$ shell vacancy from the $n=3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\mathrm{KE}+\mathrm{PE}$	mechanical energy
KE_e	kinetic energy of an ejected electron
$\mathrm{KE}_{\mathrm{rel}}$	relativistic kinetic energy
$\mathrm{KE}_{\mathrm{rot}}$	rotational kinetic energy
KE	thermal energy

Symbol	**Definition**
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_α	x rays created when an electron falls into an $n = 2$ shell from the $n = 3$ shell
L_e	electron total family number
L_μ	muon family total number
L_τ	tau family total number

Symbol	**Definition**
L_{f}	heat of fusion
L_{f} and L_{v}	latent heat coefficients
$\mathrm{L}_{\mathrm{orb}}$	orbital angular momentum
L_{s}	heat of sublimation
L_{v}	heat of vaporization
L_z	*z* - component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification

Symbol	**Definition**
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m\left({}^{A}\mathrm{X}\right)$	atomic mass of a nuclide
MA	mechanical advantage
m_{e}	magnification of the eyepiece
m_e	mass of the electron
m_ℓ	angular momentum projection quantum number

Symbol	**Definition**
m_n	mass of a neutron
m_{o}	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_{s}	spin projection quantum number
N	magnitude of the normal force
N	newton
$\mathbf{N}$	normal force
N	number of neutrons
n	index of refraction

Symbol	**Definition**
n	number of free charges per unit volume
N_A	Avogadro's number
N_r	Reynolds number
$\mathrm{N \cdot m}$	newton-meter (work-energy unit)
$\mathrm{N \cdot m}$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
$\mathbf{p}$	momentum

Symbol	**Definition**
p	momentum magnitude
p	relativistic momentum
$\mathbf{p}_{\text{tot}}$	total momentum
$\mathbf{p}'_{\text{tot}}$	total momentum some time later
P_{abs}	absolute pressure
P_{atm}	atmospheric pressure
P_{atm}	standard atmospheric pressure
PE	potential energy
PE_{el}	elastic potential energy
PE_{elec}	electric potential energy

Symbol	**Definition**
PE_{s}	potential energy of a spring
P_{g}	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge

Symbol	**Definition**
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
R	antired (cyan) quark color
R	resistance
R	resultant or total displacement

Symbol	Definition
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man

Symbol	**Definition**
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the *n*th H-atom orbit
R_{p}	total resistance of a parallel connection
R_{s}	total resistance of a series connection
R_{s}	Schwarzschild radius
S	entropy
$\mathbf{S}$	intrinsic spin (intrinsic angular momentum)

Symbol	**Definition**
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
$\mathbf{s}$	total displacement
$\sec\theta$	secant
$\sin\theta$	sine
s_z	*z*-component of spin angular momentum

Symbol	Definition
T	period—time to complete one oscillation
T	temperature
T_c	critical temperature—temperature below which a material becomes a superconductor
T	tension
T	tesla (magnetic field strength *B*)
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan\theta$	tangent
U	internal energy

Symbol	Definition
u	quark flavor up
u	unified atomic mass unit
$\mathbf{u}$	velocity of an object relative to an observer
$\mathbf{u}'$	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
$\mathbf{v}$	relative velocity between two observers
v	speed of light in a material

Symbol	**Definition**
$\mathbf{v}$	velocity
$\mathbf{v}$	average fluid velocity
$V_{\mathrm{B}} - V_{\mathrm{A}}$	change in potential
$\mathbf{v}_{\mathrm{d}}$	drift velocity
V_{p}	transformer input voltage
V_{rms}	rms voltage
V_{s}	transformer output voltage
$\mathbf{v}_{\mathrm{tot}}$	total velocity
v_{w}	propagation speed of sound or other wave
$\mathbf{v}_{\mathrm{w}}$	wave velocity

Symbol	Definition
W	work
W	net work done by a system
W	watt
w	weight
w_{fl}	weight of the fluid displaced by an object
W_{c}	total work done by all conservative forces
W_{nc}	total work done by all nonconservative forces
W_{out}	useful work output
X	amplitude
X	symbol for an element

Symbol	**Definition**
${}^{Z}_{A}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
X_{C}	capacitive reactance
X_{L}	inductive reactance
x_{rms}	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)

Symbol	Definition
Z	impedance